Constructing Minimum Energy Mobile Wireless Networks

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Energy conservation is a critical issue in designing wireless ad hoc networks, as the nodes are powered by batteries only. Given a set of wireless network nodes, the directed weighted transmission graph G_t has an edge uv if and only if node v is in the transmission range of node u and the weight of uv is typically defined as $||uv||^\alpha + c$ for a constant $2 \leq \alpha \leq 5$ and $c > 0$. The minimum power topology G_m is the smallest subgraph of G_t that contains the shortest paths between all pairs of nodes, i.e., the union of all shortest paths. In this paper, we described a distributed position-based networking protocol to construct an enclosure graph G_e, which is an approximation of G_m. The time complexity of each node u is $O(\min(d_{G_t}(u)d_{G_e}(u), d_{G_t}(u)\log d_{G_t}(u)))$, where $d_G(u)$ is the degree of node u in a graph G. The space required at each node to compute the minimum power topology is $O(d_{G_t}(u))$. This improves the previous result that computes G_m in $O(d_{G_t}(u)^3)$ time using $O(d_{G_t}(u)^2)$ spaces. We also show that the average degree $d_{G_e}(u)$ is usually a constant, which is at most 6. Our result is first developed for stationary network and then extended to mobile networks.

I. Introduction

Mobile wireless networking has received significant attention over the last few years due to their wide potential applications in various situations such as battlefield, emergency relief and etc. [17, 19]. There are no wired infrastructures or cellular networks in *ad hoc* wireless network. Mobile nodes communicate with each other either through a single-hop transmission if the receiver node is within the transmission range of sender, or through multi-hop wireless links by using intermediate nodes to relay the message. In other words, each node in the network also acts as a router, forwarding data packets for other nodes. A single transmission by a node can be received by all nodes within its transmission range. There are two models of the transmission range of all nodes: either all nodes have the same transmission power, or each node can adjust its transmission power independently according to its neighborhood information to possibly reduce the energy consumption. In this paper, we always assume that each mobile node can adjust the transmission power accordingly. Consequently, even a node v is within the transmission range of another node u, it may be energy efficient to use another node to relay the signal sent from u to v. Each mobile node typically has a portable set with transmission and reception processing capabilities. In addition, we assume that each node has a low-power GPS receiver, which provides the position information of the node itself. There are also several other means to get the relative or absolute position of wireless node. One way is to use signal strength and direction.

In this paper, we model a wireless network by a weighted directed graph $G_t = (V, E)$. Here V is the set of all mobile nodes, and edge $(u, v) \in E$ if and only if the node v is in the transmission range of the node u. The weight of the edge (u, v) is the power consumed for transmitting signal from u to v and possibly the energy consumed by node v to process the received signal. Hereafter, we call G_t the *transmission graph*. When all nodes have the same transmission range, the transmission graph is often called the *unit disk graph*, whose properties were studied extensively. We assume that G_t is *strongly connected*. Here a graph is strongly connected if there is a directed path from any node to any other node.

A central challenge in the design of *ad hoc* networks is the development of dynamic routing protocols that can efficiently find energy-efficient routes between two communication nodes. In recent years, a variety of routing protocols [2, 13, 14, 15, 16, 20] targeted specifically for *ad hoc* environment have been developed. For the review of the state of the art routing protocols, see surveys by Royer and Toh [19] and by Ramanathan and Steenstrup [17]. Energy conservation is a critical issue in *ad hoc* wireless network for the node and network life, especially in the sensor networks, as the nodes are powered by batteries only. Thus, it is almost imperative to find energy-efficient routes to conserve energy. Notice that there are two different objectives in minimizing the consumed energy. One is to minimize the energy used by all nodes evolved in one communication session. The other objective is to make the network life as longer as possible. In this paper, we concentrate on constructing a subgraph of the transmission graph such that we could still find a path conserving energy for one communication session. It is not the intension of this paper to find this path, which can be done by shortest path algorithm or be approximated by some other algorithms. Entending the network life involves the global scheduling of different routings, which was also studied extensively [3, 10].

In this paper, we consider how to find a distributed networking protocol optimized for achieving the minimum energy for randomly deployed ad hoc networks. A directed path from a node s to a node t is said to be the *minimum-power path* if it consumes the least power among all paths from s to t. Then, our objective is to find the minimum directed subgraph $G_m = (V, E)$ of G_t, which is union of the minimum-power paths between any pair of nodes. Hereafter, the graph G_m is also called *minimum-power topology*. Given a node u, call a node v a *neighbor* of u if there is no power efficient two-hops relay for the signal from u to v. In other words, edge uv is the minimum-power path for pair of nodes u and v. Thus, graph G_m contains all such edges.

Rodoplu and Meng [18] described a distributed protocol to find an application of the minimum-power topology for a stationary ad hoc network and discussed possible extensions to the dynamic network. Their algorithm finds the topology via a local search in each nodes surrounding. Each mobile node u first finds all nodes, denoted by $T(u)$, lied in its transmission range. The node u then tries to find nodes in $T(u)$ such that it can not be the neighbor of u. However, their protocol is not time and space efficient. The worst time complexity could be$O(d_{G_t}(u)^3)$, where $d_{G_t}(u)$ is the number of nodes lied in the transmission range of u. Moreover, the possible space required by node u is $O(d_{G_t}(u)^2)$.

In this paper, we described a distributed position-based networking protocol optimized for minimum energy consumption in mobile networks. Instead of constructing the minimum-power topology G_m, we construct an enclosure graph G_e, which contains G_m but not much larger. The enclosure graph is formed by connecting each node to its neighbors. Each mobile node u, instead of finding nodes that can not be served as relay nodes [18], tries to find the nodes that are guaranteed to be the neighbors of u. The proposed protocols are more time-efficient and space-efficient than that proposed by Rodoplu and Meng [18]. The time complexity of each node u is $O(\min(d_{G_t}(u)d_{G_e}(u), d_{G_t}(u)\log d_{G_t}(u)))$ when $\alpha = 2$ or $c = 0$, where $d_G(u)$ is the degree of node u in a graph G. We also show that in most case, the average degree $d_{G_e}(u)$ is usually a constant by showing that the graph G_e is a subgraph of the Delaunay triangulation of all mobile nodes. The space required at each node u is $O(d_{G_t}(u))$. Our result is first developed for stationary network and then extended to mobile network.

The rest of the paper is organized as follows. In Section II, we first review some preliminary definitions and results related to the minimum energy topology and enclosure graph. In Section III, we study the basic properties of the neighbors of a node and the enclosure graph. Our distributed algorithm is proposed in Section IV and it is then extended for dynamic wireless ad hoc network in section V. We conclude our paper in Section VI.

II. Preliminaries

A major focus of this work is to design a networking protocol that is self-configuring and to construct a sparse *supergraph* of all shortest paths between all pairs of nodes. We consider the true peer-to-peer case, in which all nodes can send messages to all other nodes.

II.A. Energy Consumption Model

In the most common power-attenuation model, the signal power falls as $\frac{1}{r^\alpha}$, where r is the distance from the transmitter antenna and α is a constant between 2 and 5 dependent on the wireless transmission environment. [1] This is typically called the *path loss*. We always assumed that all receivers have the same power threshold for signal detection, which are then typically normalized to one. The path loss normally depends on the heights of the transmit antennas as

[1] To make this model meaningful, we always assume that the distance between any two nodes is at least one unit so the above model does not violate the energy-consumption law. We also assume that the unit of power and the unit of distance between nodes satisfies the path loss formula.

well as the transmitter-receiver separation. In this paper, we assume that all mobile devices have similar antenna heights so that we will only concentrate on path loss that is distance-dependent. With these assumptions, the power required to support a link between two nodes u and v separated by distance r is r^{α}, which is called the *transmitter power* of node u. By a simple geometry computing, it is easy to see that relaying signal between nodes may result in lower power consumption than communication over a large distances due to the non-linear power attenuation. For convenience, hereafter, we use $||uv||$ to denote the Euclidean distance between two geometry nodes u and v.

As a simple illustration, consider three nodes s, r, and d on the plane as in Figure 1. Assume that all three nodes use identical transmitters and receivers and $\alpha = 2$. The power to transmit a signal from s to d is therefore $||sd||^2$. If we use the node r to relay the signal, the total power used is $||sr||^2 + ||rd||^2$, which is less than $||sd||^2$. In other words, if s wants to send a messages to any node d lying in the right side of the line l, relaying through node r always consumes less power than directly transmitting to d.

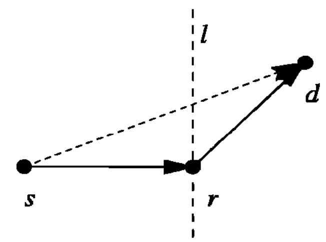

Figure 1: Relaying through other node r consumes less power than directly transmitting from s to d.

If only path loss is considered, recently, Li *et al.* [9, 11] described several methods that generate the minimum-power topology with theoretical guarantee. For example, they showed that the Gabriel graph always contains the minimum-power topology. Li [8] also show that the localized Delaunay triangulation contains the minimum-power path; moreover, given any pair of nodes, it contains a path whose length is no more than 2.5 times length of the shortest path connecting them. He also show that localized Delaunay triangulation is a planar graph.

There is also another source of power consumption we must consider in addition to the path loss. When a node receives a signal from other node, it needs consume some power to receive, store and then process that signal [18]. This additional power consumed at the receiver node is referred as the *receiver power* at the relay node. Typically, every relay node consumes the same receiver power due to the nature of its operations. Hereafter, we will denote such power by a constant c. Notice that additional power will also be consumed when running the routing algorithm. In the design of modern processors, however, the power consumption required for such processing and computation can be made negligible compared to the transmitter power and receiver power.

In this paper, we assume that the mobile nodes are given as a finite point set $\mathcal{V}$ in a two-dimensional plane. Let n be the number of mobile nodes. Consider any unicast π from a node $u \in \mathcal{V}$ to another node $v \in \mathcal{V}$:

$$\pi = p_0 p_1 \cdots p_{m-1} p_m, \ where \ u = p_0, \ v = p_m.$$

The total transmission power consumed by this path π is

$$\sum_{i=1}^{m} ||p_{i-1}p_i||^{\alpha} + m \cdot c.$$

We define a weighted transmission graph G_t over all nodes of $\mathcal{V}$, and the weight of an edge $p_i p_j$ is equal to $||p_i p_j||^{\alpha} + c$. Then the minimum-power path can be computed by applying any shortest path algorithm such as Dijkstra's algorithm. The cost of the centralized Dijkstra's algorithm is $O(n \log n + E)$, where E is the number of edges of the graph G_t. Notice that the transmission graph could have $O(n^2)$ edges. This implies that it needs $O(n^2)$ to compute the shortest path by applying Dijkstra's algorithm on G_t. Later, we will show that it is sufficient to apply the shortest path algorithm on an enclosure graph G_e (defined later), which is usually a sparse graph. Thus it is more time efficient to compute the minimum-power routing using the enclosure graph.

II.B. Network Model

A wireless network is modelled by a directed transmission graph $G_t = (V, E)$. Let $T(u)$ be the set of nodes within the transmission range of node u. Thus, all nodes in $T(u)$ can receive the message transmitted by node u, and then, can serve as the relay nodes. However, it may be not power-efficient to use all nodes in $T(u)$ to directly relay the message from u to other nodes. [2] For example, let's consider the following simple configuration of three mobile nodes u, r, d illustrated by the Figure 1. Let $\alpha = 2$. Assume that nodes r and d are within the transmission range of the node u and $||ur||^2 + ||rd||^2 + c \leq ||ud||^2$. Then

[2] Here a node v *directly* relays the signal from a node u if u sends signal to v and v then relay it.

it is power efficient to use node r to relay the signal from u to node d than transmitting directly to node d. It implies that node d can never be used to directly relay the signal from node u. As shown later, the average number of nodes that can directly relay the signal from a node is at most 6 for most situations.

Based on the observation concerning relays, we will first consider how to find the minimum-power topology in a wireless network where all nodes are assumed to be stationary. For example, the sensors of sensor-based wireless network has little movement. Our main goal is to develop an algorithm that requires mainly local computation for computing and updating the topology. From the perspective of the power consumption, a distributed algorithm running almost exclusively on local information requires the transmission only over small distances. It also dramatically reduces the interference levels among nodes. We separately study the following cases: the receiver's cost c is negligible; the propagation environment constant $\alpha = 2$, and the general cases $c > 0$ and $2 < \alpha \leq 5$.

II.C. Basic Definitions

First let us study a simple case. Assume that node s want to send a message to node d. Accordingly, node s is called the source (transmitter) and node d is called the destination (receiver). A node r could be used as a relay node if and only if

$$||sd||^\alpha + c > ||sr||^\alpha + c + ||rd||^\alpha + c.$$

Notice that $||sd||^\alpha + c$ is the power incurred if node s directly transmits signal to node d, and $||sr||^\alpha + c + ||rd||^\alpha + c$ is the power incurred if node s uses the node r as the relay node for transmission from s to node d. Thus, given node s and node r, the locus of all node d, such that relaying through node r consumes less power than directly transmitting from s to d, is called the *relay region* of r for s [18]. Hereafter, we denote such relay region as $R_{\alpha,c}(s,r)$. When it is clear from the context, we will drop the α and/or c from $R_{\alpha,c}(s,r)$.

Definition 1 [RELAY REGION] *The relay region of a node r for a node s is defined as*

$$R_{\alpha,c}(s,r) = \{x \mid ||sx||^\alpha > ||sr||^\alpha + ||rx||^\alpha + c\}.$$

Figure 2 illustrates typical relay regions in propagation environment with $\alpha = 2$ and $\alpha = 4$ respectively. We then study in detail what is the mathematical formula to represent the relay region $R_{\alpha,c}(s,r)$. Let (x_p, y_p) denote the position of a two-dimensional

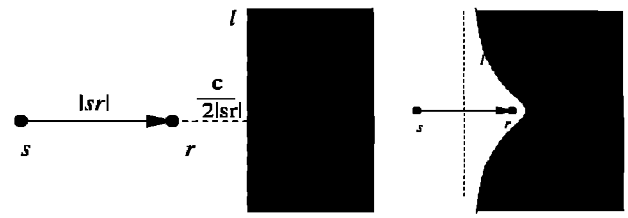

Figure 2: The relay regions $R(s,r)$ denoted by the shaded area. Left $\alpha = 2$; right $\alpha = 4$.

node p. Assume that node r has coordinates $(0,0)$ and node s has coordinates $(-||sr||, 0)$. When $\alpha = 2$, $d = (x_d, y_d) \in R(s,r)$ implies that

$$\begin{aligned} ||sd||^2 &= (||sr|| + x_d)^2 + y_d^2 \\ &> ||sr||^2 + ||rd||^2 + c \\ &= ||sr||^2 + x_d^2 + y_d^2 + c. \end{aligned}$$

It implies that $x_d > \frac{c}{2||sr||}$. In other words, if $s = (-||sr||, 0)$ and $r = (0,0)$, then

$$R_{2,c}(s,r) = \{(x,y) \mid x > \frac{c}{2||sr||}\}.$$

Therefore, the boundary of the relay region $R_{2,c}(s,r)$ for any two nodes s and r is a line perpendicular to sr and node r has distance $\frac{c}{2||sr||}$ to the relay region. See the above Figure 2 for an illustration. When $\alpha = 4$, we have

$$\begin{aligned} ||sd||^4 &= ((||sr|| + x_d)^2 + y_d^2)^2 \\ &= (||sr||^2 + x_d^2 + y_d^2 + 2x_d||sr||)^2 \\ &> ||sr||^4 + ||rd||^4 + c \\ &= ||sr||^4 + x_d^4 + y_d^4 + 2x_d^2 y_d^2 + c. \end{aligned}$$

It implies that

$$(2x_d + ||sr||)y_d^2 + 2x_d^3 + 3||sr||x_d^2 + 2||sr||^2 x_d > \frac{c}{2||sr||}.$$

Given any two nodes s and r, let h_{rs} be the half plane defined by the bisection line of segment sr and containing node r. Then it is easy to show that the relay region $R_{\alpha,c}(s,r)$ is inside h_{rs} for any propagation environment constant α and receiver cost c.

Remark 1 *The fact that a node d is in the relay region $R(s,r)$ does not imply that node r has to relay the signal to d. The fact that a node d is not in the relay region $R(s,r)$ does not imply that node r will not relay the signal to d. It is not difficult to construct examples to verify these.*

We then study the properties of the structure of the minimum energy topology of a set of stationary nodes. For simplicity, let $E_{\alpha,c}(s,r)$ be the complement of $R_{\alpha,c}(s,r)$. The region $E_{\alpha,c}(s,r)$ is called the enclosure region of node s by node r.

Definition 2 [ENCLOSURE REGION] *The enclosure region $E_{\alpha,c}(s)$ of a node s is defined as*

$$E_{\alpha,c}(s) = \bigcap_{r \in T(s)} E_{\alpha,c}(s,r).$$

Notice that the above definition is analog to the *Voronoi region* of a node s, which is defined as

$$V(s) = \{x \mid \forall q \in \mathcal{V},\ ||xs|| < ||xq||\}.$$

Remember that here $T(s)$ is the set of nodes lying within the transmission range of node s.

A node u is said to a *neighbor* of a node s if it is inside the enclosure region $E_\alpha(s)$ of node s. Figure 3 shows an example of the enclosure region and the neighbors of a node s.

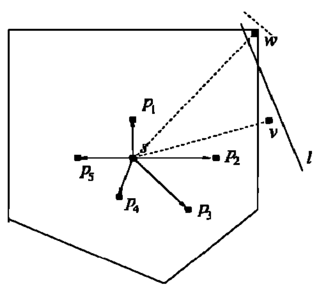

Figure 3: The enclosure region and the neighbors of a node when $\alpha = 2$. Here $N(s) = \{p_1, p_2, p_3, p_4, p_5\}$.

Definition 3 [NEIGHBORS] *The neighbors $N_{\alpha,c}(s)$ of a node s is defined as*

$$N_{\alpha,c}(s) = \{u \mid u \in T(s),\ u \in E_{\alpha,c}(s)\}.$$

When it is clear from the context, we will also drop the constant α and/or c from $E_{\alpha,c}(s,r)$ and $N_{\alpha,c}(s)$. Notice that in [18] they defined the enclosure region as

$$\widetilde{E}_\alpha(s) = \bigcap_{r \in \widetilde{N}_\alpha(s)} E_\alpha(s,r)$$

and define the neighbor as

$$\widetilde{N}_\alpha(s) = \{u \mid u \in \mathcal{N} \ and \ u \in \widetilde{E}_\alpha(s)\}.$$

Unfortunately, there are discrepancy in the their definitions of enclosure region and neighbor. Consider an example illustrated in Figure 3. The node v is not in the neighbor set $N_\alpha(s)$ of node s because it is power efficient to use the node p_2 to relay the message from s to v. However, we need node v to define the enclosure region. Thus, node w is not the neighbor of node s. However, by the definition of [18], the set $\{p_1, p_2, p_3, p_4, p_5, w\}$ is also a feasible solution for $\widetilde{N}_\alpha(s)$. A main observation here is that even a node does not affect $E_\alpha(s)$, it is still possible that this node is inside $E_\alpha(s)$ and thus it is the neighbor of s. For example, in Figure 3, if node v does not exist, then node w is a neighbor of s. As [18], we define the enclosure graph as following.

Definition 4 [ENCLOSURE GRAPH] *The enclosure graph $G_e^{(\alpha,c)} = (V, E)$ of a set of mobile nodes $\mathcal{V}$ is the directed graph whose vertices are $\mathcal{V}$ and whose edges are all (u,v), where $v \in N_{\alpha,c}(u)$.*

When it is clear from the context, we will also drop the α and/or c from $G_e^{(\alpha,c)}$. Then we proved that the minimum energy topology is always contained in the enclosure graph.

Theorem 1 [MAIN THEOREM] *The enclosure graph G_e contains the minimum-power topology.*

PROOF. Consider any two nodes s and d. Let path $\pi = sv_1 \cdots v_m d$ be the minimum energy path from s to d. Then it is obvious that we can not use any other node to relay the signal from s to v_1. It implies that v_1 is in the neighbors $N(s)$ of node s. Consequently, the enclosure graph $G_{\alpha,c}$ contains the minimum energy topology. ◻

Notice that it is not difficult to construct an example such that the enclosure graph is not equal to the minimum energy topology. Figure 4 shows such an example when $\alpha = 2$ and $c = 0$. It is not difficult to show that edge uv does not belong to the minimum-power topology because we have a path $uxyv$ consuming less power. However, as we will showed later, usually the number of edges in G_e is $O(n)$.

III. Properties of the Minimum Power Network

In this section, we review and develop some mathematical theories studying the general properties of the enclosure graph and the minimum-power topology on stationary ad hoc network.

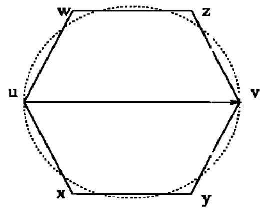

Figure 4: The enclosure graph and the minimum-power topology is not same: the drawn graph is enclosure graph while the minimum-power topology does not have uv.

III.A. General Properties

It is shown in [18] that the relay region satisfies the following properties.

Lemma 2 *Relay regions satisfy the following properties.*

1. *for any node u, $u \notin R(s,u)$*
2. *for any two nodes u, v, if $u \in R(s,v)$ then $v \notin R(s,u)$*
3. *for any three nodes u, v, w, it is impossible that $u \in R(s,v)$, $w \in R(s,u)$ and $v \in R(s,w)$ are all satisfied.*

If a node $u \in R(s,v)$ then $||su|| > ||sv||$ and $||su|| > ||uv||$. Then it is easy to show the correctness of the above lemma. However, it is possible there are $m > 3$ nodes $p_1, p_2, \cdots, p_{m-1}, p_m$ such that $p_{i+1} \in R(p_{i-1}, p_i)$ for all $1 \leq i \leq m$. Here $p_0 = p_m$ and $p_{m+1} = p_1$. For example, let us consider six nodes that form a hexagon $p_1p_2p_3p_4p_5p_6$. Here the side length of the hexagon is $\sqrt{c} + \epsilon$ and $\alpha = 2$. Then it is easy to show that $p_{i+1} \in R(p_{i-1}, p_i)$ for all $1 \leq i \leq 6$. If the side length of the hexagon is less than $\sqrt{c}/2$, then the enclosure graph defined on these 6 nodes is a completed graph.

Lemma 3 *Let node v be the nearest neighbor of node u. Then $v \in N_\alpha(u)$.*

PROOF. Assume that $v \notin N_\alpha(u)$. In other words, it is power efficient to relay the signal from node u to node v through some other nodes. Assume that a sequence of node $v_1, v_2, \cdots, v_m$ is used to relay. It is obvious that the total power consumed by path $uv_1v_2\cdots v_mv$ is at least $||uv_1||^\alpha + c$, which is at least $||uv||^\alpha + c$. It is a contradiction that path $uv_1v_2\cdots v_mv$ consumes less power than directly transmitting from u to v. Therefore $v \in N_\alpha(u)$. □

III.B. Without Receiver Cost

We then study the situation that the receiver's cost is negligible compared to the transmission cost incurred. In [23], Wan *et al.* presented several approximations algorithm for constructing the broadcast tree such that the total energy consumed by all nodes is minimized when there is no receiver's cost. In this paper, we constructed a networking graph that contains the minimum-power path between any pair of nodes.

Lemma 4 *Given two nodes s and r, the relay region $R_{\alpha_1}(s,r) \subset R_{\alpha_2}(s,r)$ if $\alpha_1 < \alpha_2$ and the receiver's cost $c = 0$.*

PROOF. Consider any point x in the relay region $R_{\alpha_1}(s,r)$. From definition 1, we know that $||sx||^{\alpha_1} > ||sr||^{\alpha_1} + ||rx||^{\alpha_1}$. And it is always true that $||sx|| > ||sr||$ and $||sx|| > ||rx||$. The fact that $||sx||^{\alpha_2} = ||sx||^{\alpha_1} \cdot ||sx||^{\alpha_2-\alpha_1}$ implies that

$$\begin{aligned} ||sx||^{\alpha_2} &> (||sr||^{\alpha_1} + ||rx||^{\alpha_1}) \cdot ||sx||^{\alpha_2-\alpha_1} \\ &= ||sr||^{\alpha_1} \cdot ||sx||^{\alpha_2-\alpha_1} + ||rx||^{\alpha_1} \cdot ||sx||^{\alpha_2-\alpha_1} \\ &> ||sr||^{\alpha_1} \cdot ||sr||^{\alpha_2-\alpha_1} + ||rx||^{\alpha_1} \cdot ||rx||^{\alpha_2-\alpha_1} \\ &= ||sr||^{\alpha 2} + ||rx||^{\alpha_2}. \end{aligned}$$

Therefore point x is also in the relay region $R_{\alpha_2}(s,r)$ by definition. This completes the proof that $R_{\alpha_1}(s,r) \subset R_{\alpha_2}(s,r)$. □

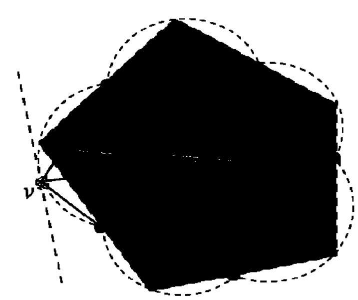

Figure 5: The Delaunay neighbors $D(u)$ and the minimum-power neighbors $N(u)$ (solid nodes). Here $D(u) - N(u) = \{v\}$.

Let *Del* $\mathcal{V}$ be the Delaunay triangulation of all mobile nodes. Here a triangulation is Delaunay if the interior of the circumcircle of each of its triangles does not contain any nodes. Let $D(u)$ be the nodes in *Del* $\mathcal{V}$ that are connected to node u. We then show that $D(u)$ contains the neighbors $N(u)$ of node u if $c = 0$. See Figure 5 for an illustration.

Lemma 5 *The neighbors $D(u)$ of a node u in the Delaunay triangulation contains $N_\alpha(u)$ for any $\alpha \geq 2$ if the receiver cost is negligible, i.e., $c = 0$.*

PROOF. It is equivalent to show that for any node in $T(u) - D(u)$, it is not inside the enclosure region $E(u)$. Consider a node $z \in T(u) - D(u)$. Assume that it is in the wedge region defined by $\measuredangle wuv$. Here triangle ∇wuv is a Delaunay triangle connected to node u. Let y be the point such that yw is perpendicular to wu and yv is perpendicular to vu. Figure 6 illustrates the proofs that followed. Then from the

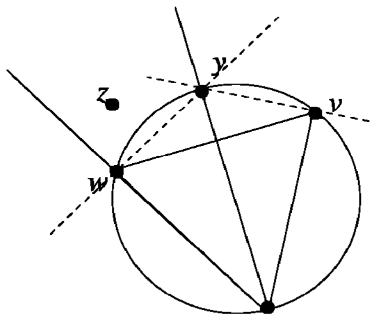

Figure 6: Any node $z \notin D(u)$ is not neighbor of u.

Delaunay property, we know that the polygon $ywuv$ does not contain any mobile nodes inside. Without loss of generality, assume that node z is inside the wedge $\measuredangle yuw$. It implies that $z \in R_2(u, w)$. From lemma 4, we know that $R_2(u, w) \subset R_\alpha(u, w)$ for any $\alpha \geq 2$. In other words, we can use node w to relay the signal from node u to node z if the receiver's cost $c = 0$. [3] Therefore, all nodes not in $D(u)$ can not be a neighbor of u. This completes the proof. ◻

The fact that the Delaunay triangulation is a planar graph implies that the enclosure graph $G_e^{\alpha,0}$ is a planar graph. Thus the number of edges is at most $3n$ if the receiver's cost is negligible. It implies that the average number of edges incident to a node u is at most 6. Notice that it is possible that the neighbors of one specific node could be as large as $O(n)$. Here all wireless nodes except a node u are on a circle and within the transmission range of node u. Then we have $d_{G_e}(u) = n - 1$.

Notice that some Delaunay neighbors $D(u)$ could be out of the transmission range of u. Let $D_T(u)$ be the Delaunay neighbors $D(u)$ that are within the transmission range of u. As shown by the previous proof, the neighbors $N(u)$ must be contained in $D_T(u)$. Moreover, it is easy to show that $D_T(u)$ is a subset of the nodes connected to u in the Delaunay triangulation of nodes $T(u)$. Recently, Li [8] proposed a new structure called the local Delaunay triangulation and showed how to construct it efficiently using linear number of messages. Li *et al.* [9, 11] showed that the Gabriel graph contains the minimum-power topology when $c = 0$. Here an edge uv belongs to the Gabriel graph if the circle using uv as a diameter does not contain any other node inside.

[3] Notice that node z is within the transmission range of u and z is contained in the wedge $\measuredangle yuw$ imply that node w is within the transmission range of u. Thus it is valid to claim that node z is in the relay region $R(u, w)$.

III.C. With Receiver Cost

Finally, we study the basic properties of the structure of the minimum-power topology if the receiver's cost is not negligible compared to the transmission cost incurred.

Lemma 6 *Given two nodes s and r, the relay region $R_{\alpha_1}(s, r) \subset R_{\alpha_2}(s, r)$ if $\alpha_1 < \alpha_2$ and the distance $||sr|| \geqslant 1$.*

PROOF. Consider any point x in the relay region $R_{\alpha_1}(s, r)$. From definition 1, we know that $||sx||^{\alpha_1} > ||sr||^{\alpha_1} + ||rx||^{\alpha_1} + c$. And it is always true that $||sx|| > ||sr||$ and $||sx|| > ||rx||$. The fact that $||sx||^{\alpha_2} = ||sx||^{\alpha_1} \cdot ||sx||^{\alpha_2 - \alpha_1}$ implies that

$$\begin{aligned}
||sx||^{\alpha_2} &> (||sr||^{\alpha_1} + ||rx||^{\alpha_1} + c) \cdot ||sx||^{\alpha_2-\alpha_1} \\
&> ||sr||^{\alpha_1}||sx||^{\alpha_2-\alpha_1} + ||rx||^{\alpha_1}||sx||^{\alpha_2-\alpha_1} \\
&\quad + c||sx||^{\alpha_2-\alpha_1} \\
&> ||sr||^{\alpha_1} \cdot ||sr||^{\alpha_2-\alpha_1} + ||rx||^{\alpha_1} \cdot ||rx||^{\alpha_2-\alpha_1} + c \\
&= ||sr||^{\alpha 2} + ||rx||^{\alpha_2} + c.
\end{aligned}$$

Therefore point x is also in the relay region $R_{\alpha_2}(s, r)$ by definition. This completes the proof. ◻

The above lemma implies that if the signal from s to a node z can not be relayed by a node r for propagation constant α_1 and $||sr|| \geq 1$, then it can not be relayed by r for any propagation constant $\alpha_2 > \alpha_1$.

III.D. Centralized Algorithm

Let us first consider how to compute the enclosure graph G_α using a centralized algorithm. One simple approach is as follows. For each pair of nodes u and v, compute the relay region $R(u, v)$ and $R(v, u)$. Then use an approach similar to computing the Voronoi diagram, we can compute the enclosure region for each node u. And all nodes covered by the enclosure region of u are connected to node u. The time complexity of the above approach could be as large as $O(n^3)$, where n is the number of all mobile nodes. This is apparently not practical for ad hoc networking.

When the receiver's cost c is negligible, we know that the minimum-power topology G_m is a subgraph of the Delaunay triangulation of all mobile nodes. Therefore, we can apply any $O(n \log n)$ time complexity Delaunay triangulation algorithm to compute the Delaunay triangulation of all mobile nodes. Then for each node u, we eliminate the nodes of Delaunay neighbors $D(u)$ that are in the relay region of other nodes from $D(u)$. The remaining nodes are $N_\alpha(u)$. Notice that the average Delaunay neighbors $D(u)$ is at most 6 implies that the average time complexity to compute $N(u)$ from $D(u)$ is constant. Therefore, the average time complexity of the above algorithm using the Delaunay triangulation to compute the enclosure graph is $O(n \log n)$ if the receiver's cost $c = 0$.

We then consider how to compute the shortest path between the source node s and the destination node t, given the enclosure graph G_e. One approach is to apply the Dijkstra's algorithm to compute the path between s and t with the minimum power consumption. The time complexity will be $O(n \log n + m)$ if the algorithm is implemented using the Fibonacci heap, where m is the number of edges of G_e. When $c = 0$, we already show that the graph G_e is a subgraph of the Delaunay triangulation of all mobile nodes. Thus, we can use the algorithm by Klein, Rao, Rauch and Subramanian [5] to compute the shortest path in linear time.

Theorem 7 *The minimum power path between any two mobile nodes can be computed in $O(n \log n)$ time using a centralized algorithm if $c = 0$.*

IV. Distributed Algorithms

In this section, we describe a distributed algorithm that finds the minimum-power topology for a set of stationary nodes. In our protocol, each node only has to consider asymptotically a constant number of nodes to construct the global minimum power paths. Before presenting our algorithm, let's first review the algorithm proposed by Rodoplu and Meng [18].

IV.A. Previous Algorithm

The algorithm proposed in [18] has two phases. The first phase of the algorithm searches the enclosure of each node by using the relay graph. The second phase of the algorithm finds the optimal links on the enclosure graph by applying the distributed Bellman-Ford algorithm. The cost metric is the power consumption which includes the transmission power cost and the receiver's power cost. Notice that it is proved that the enclosure graph contains the minimum power topology. When searching for neighbors using their algorithm, a node u must keep track of whether a node just found is in the relay region of previously found nodes. Let $T(u)$ be all nodes found by a node u so far. Node u stores a *relay graph* defined as follows. Whenever a node $v \in T(u)$ is in the relay region of a node $w \in T(u)$, it forms a directed edge (w, v). Then the relay graph of a node u is defined as the directed graph with all such edges on all vertices $T(u)$. It is easy to show that there is no directed cycle in the relay graph by the relay properties.

As described in [18], a new found node v by the node u is marked *alive* if it is not in the relay region of any previous found node by u. Otherwise, assume that the node v is in the relay region of a previously found node w, then they say that node w *blocks* node v. And the new found node v is marked *dead*. If node v blocks a previously marked alive node w, node w is also marked *dead*. When a previously alive node w is marked dead, it is possible that some node z blocked by w is not blocked by any other alive nodes. In other words, node z should be an alive node. They [18] then use an operation to *revive* all such nodes z that are previously marked dead. Consequently, their algorithm finds the largest subset of nodes such that any node in the subset is not blocked by any other node in the subset.

Even the basic idea of their algorithm is correct, their algorithm is, however, not time efficient and uses too much storage. The reason are as follows. First it is not space efficient and time efficient to use the relay graph to compute the neighbors. The relay graph could be very large compared to the actual number of neighbors (which is a constant in average when $c = 0$). Second, the unnecessary revive operations also waste time. Notice that we had shown that any node w lying in the relay region $R(s, r)$ can not be a neighbor of s. In next, we present a new algorithm, which is time efficient and uses only a small amount of storage.

IV.B. Our Algorithms

IV.B.1. Neglect the receiver's cost

We first consider the case when the receiver's cost could be neglected. Consider a node u in the transmission graph G_t. Remember that $d_{G_t}(u)$ is the number of discovered nearby nodes of u. We showed that the neighbors $N(u)$ is a subset of the Delaunay neighbors $D(u)$ if $c = 0$. The Delaunay neighbors $D(u)$ can also be computed efficiently by using the Voronoi diagram. In other words, instead of computing the Delaunay neighbors $D(u)$, we compute the Voronoi region of node u.

For any node v in $T(u)$, let v' be the midpoint of the line uv. We call v' the *image* of v. We will use such image point v' to compute the Voronoi region of u. Let $l_{v'}$ be the line that passes point v' and is perpendicular to the segment uv'. Let $h_{v'}$ be the half plane defined by line $l_{v'}$ containing the node u. See Figure 7 for an illustration. Then the Voronoi diagram

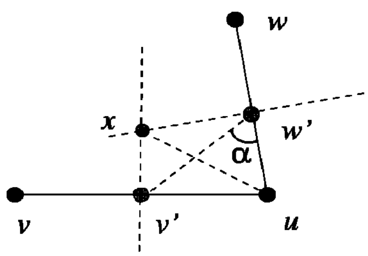

Figure 7: The Voronoi region of a local node u.

$V(u)$ of a node u is

$$\cap_{v \in T(u)} h_{v'}.$$

Assume a node v defines a segment pq in the Voronoi region of u. Then it is easy to show that points p and q can be computed in $O(d_{G_t}(u))$ time by computing the intersection points of line $l_{v'}$ with all other half planes $h_{w'}$ defined by other nodes w.

Notice that the nearest neighbor node v of u always defines a segment, say q_0q_1, in the Voronoi cell of $V(u)$. The segment q_0q_1 can be computed in $O(d_{G_t}(u))$ time. Assume point q_1 is the intersection of the line $l_{x'}$ and $l_{v'}$. Then we know that line $l_{x'}$ also defines a segment, say q_1q_2 in the Voronoi cell of $V(u)$, which can also be computed in $O(d_{G_t}(u))$ time. The following Figure 8 illustrates the above proofs. Then we can repeat the above procedure until

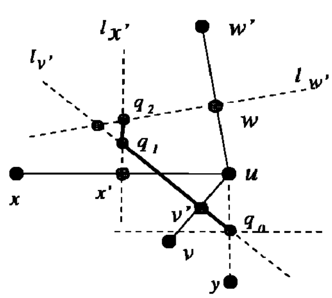

Figure 8: Comptuing the Voronoi region of a local node u.

the Voronoi region of u is computed.

Lemma 8 *The Voronoi cell of a node u can be computed in $O(d_{G_t}(u)d_{G_e}(u))$ time by node u where $d_{G_t}(u)$ is the number of nodes known by u, $d_{G_e}(u)$ is the number of segments of the Voronoi cell of u.*

Notice that $N(u) \subseteq D(u)$ when the receiver's cost could be neglected. Using the duality of the Delaunay triangulation and the Voronoi region, the above procedure also implies that $D(u)$ can be computed by u in $O(d_{G_t}(u)d_{G_e}(u))$ time. Typically the average number of Delaunay neighbors of node u is at most 6. Thus $N(u)$ can be quickly computed from $D(u)$. Notice that when $d_{G_e}(u)$ is larger than $\log d_{G_t}(u)$, we can apply the Delaunay triangulation algorithm to compute the Delaunay triangulation of all nodes within the transmission range of node u. Then extract the Delaunay neighbors of node u in $O(d_{G_e}(u))$ time. Therefore the total time complexity of this approach is $O(d_{G_t}(u)\log d_{G_t}(u))$. Notice that there is no need to compute the enclosure graph from the Delaunay triangulation because the Delaunay triangulation and the minimum power topology both have $O(n)$ edges. This implies the following lemma.

Lemma 9 *When the receiver's cost $c = 0$, we can find $N(u)$ in $O(\min(d_{G_t}(u)D(u), d_{G_t}(u)\log d_{G_t}(u)))$ time, where $D(u)$ is the number of Delaunay neighbors of u.*

Although Li *et al.* [9, 11] recently showed that Gabriel graph always contains the minimum-power topology, which can be constructed efficiently in a localized manner, we think that constructing the localized Voronoi diagram efficiently itself is worth of study. Moreover, this method can be used to find an approximation of the minimum-power topology when $c \neq 0$. Notice that, the general structure of the minimum-power topology for $c \neq 0$ is still unknown.

IV.B.2. The constant $\alpha = 2$

Notice that the above procedure to compute the Voronoi diagram of a node u can also be used to compute the neighbors $N(u)$ of node u when $\alpha = 2$ and $c > 0$. We already showed that the boundary of $R(u, v)$ is a line when $\alpha = 2$. Thus, the relay region of a node is the intersection of some half planes. Instead of defining v' as the midpoint of segment uv, we define v' as the intersection point of the line uv and the boundary of the relay region $R(u, v)$. Then similarly, we know that the neighbors $N(u)$ of a node u can be computed in $O(d_{G_t}(u)d_{G_e}(u))$ time if the propagation constant $\alpha = 2$. Figure 9 illustrates the definition of function $'$.

When the number of neighbors of u is more than $\log d_{G_t}(u)$, we also use the Delaunay triangulation to find the neighbors as follows. For each node v, it is

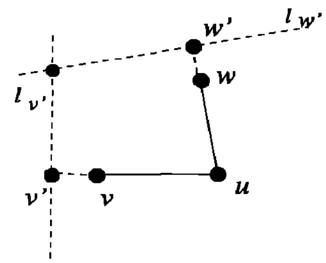

Figure 9: The modified Voronoi region of a local node u when $alpha = 2$.

mapped to a point v' as the intersection point of line uv and the boundary of $R(u, v)$. Define point v'' such that v' is the midpoint of segment uv'. Then compute the Delaunay triangulation χ of the point set $\{v'' \mid v \in T(u)\} \cup \{u\}$. The neighbors of node u in the Delaunay triangulation χ is then $N(u)$. This procedure has time complexity$O(d_{G_t}(u) \log d_{G_t}(u))$. Consequently, the following lemma is straightforward.

Lemma 10 *The neighbors $N(u)$ of node u can be computed in $O(\min(d_{G_t}(u) d_{G_e}(u), d_{G_t}(u) \log d_{G_t}(u)))$ time if the propagation constant $\alpha = 2$.*

IV.B.3. General Cases

Notice that the approach used to compute the Voronoi region can also be used to compute the enclosure region of a node u generally but the definition of the bisector is different. For general α and constant c, the bisector is the boundary of relay region $R(u, v)$. The time complexity to compute the enclosure region $E(u)$ is $O(d_{G_t}(u) d_{G_e}(u))$. However, actually it could be expensive. The most expensive operation will be computing the intersection point of the boundaries of two relay regions. Instead, we propose to use the following method. For each node u, its nearest neighbor is always in $N(u)$. Here a node v is the nearest neighbor of u if v is the closest node to u geometrically besides u itself. Remember that $T(u)$ is the set of all nodes that are within the transmission range of node u.

Algorithm 1 *Min-Power Topology(u)*

1. $N'(u) = \phi$; $Q = T(u)$.
2. while $(Q \neq \phi)$ {
3. Let $v \in Q$ be the nearest node to u;
4. $N'(u) = N'(u) \cup \{v\}$;
5. Eliminate all nodes x from Q such that $||uv||^\alpha + ||vx||^\alpha > ||ux||^\alpha + c$; }

Notice that the computed result $N'(u)$ is guaranteed to contain $N(u)$. If the computed set $N'(u)$ is still large, we can apply the above method on $N'(u)$ to refine the solution. Here Q is the set of all possible neighbor nodes within the transmission range of u. We find the nearest neighbor node v of u from Q and add it to $N(u)$. By the definition of the enclosure region and the neighbors, we know that all nodes from $Q \cap R(u, v)$ could not be the neighbors of u. Then we can eliminate them first. The above procedure is repeated until Q is empty. Notice that each node will be eliminated once or put into $N'(u)$. Thus the main complexity comes from searching the nearest node of u from Q. It is easy to show that the time complexity of the above algorithm is $O(d_{G_t}(u) \log d_{G_t}(u))$, if we sort the distance of all nodes from $T(u)$ to u.

It is not difficult to show that the above method (Algorithm 1) actually computes the one-hop relay neighbors. The one-hop relay neighbors of a node s is the set of all nodes v such that path svd is more power efficient than path sd for some node d. Recently, Li *et al.* [7] proposed to consider all k-hop paths. A node v is a neighbor of a node s if there is no a path with k-hops such that it consumes less power than edge sv.

After the network topology is constructed, we assign the cost of each link (u, v) as $||uv||^\alpha + c$ and store the cost at node u. Then, given a master-site s, we can use the distributed Bellman-ford algorithm to find the shortest path tree rooted at s. However, as the dynamic nature of the mobile wireless network, we argue that it is not necessary to compute the tree and store the structure at each node. The reasons are as follows. First the shortest path tree is computed for the master-site. It is not on-demand for any other node. Second, typically, the traffic is often unicast from one node to the other node. So it wastes the resources to compute the shortest path tree which is not required even the sender is the master-site. Third, because of the dynamic moving of the mobile nodes, the shortest path tree is often out-dated after some short time period. Therefore, our algorithm mainly concentrate on building the networking topology such that it contains the shortest paths from any node to any other node. Each traffic session initiate the on-demand routing protocol from the source to the destination. For more on-demand routing, see [4, 12, 15, 17]. We also can use the compass routing [1, 6] or Voronoi region based routing [21] to route the packet from a node s to a node t using this enclosure graph. Notice that the compass routing and the Voronoi region based routing can not guarantee that the computed route consumes the minimum energy. However, it is much faster than the optimal Bellman-ford algorithm.

IV.B.4. Setting the Search Region

Notice that in the algorithm proposed by Rodoplu and Meng [18], it is not specified when to stop exploring new nodes that could be the neighbors of a node u. They recongnized that it is challenging to find such search region such that the energy consumption until the algorithm terminates is minimized. By using our localized approach, we have a simple criteria for stopping exploring new nodes. We find new nodes using the following sequences of transmission powers p, $2p$, $4p$, $\cdots$, $2^i p$, $\cdots$. Here p is a predefined constant transmission power. We stop transmitting using power $2^i p$ if the enclosure region computed by using nodes found so far is in the circle centered at u with radius $(2^{i-1}p)$. Let $B(u,r)$ be the disk centered at u with radius r. Let $E_r(u)$ be the enclosure region of node u computed using all nodes within disk $B(u,r)$. Then the following lemma supports our algorithm.

Lemma 11 *If $E_r(u)$ is inside disk $B(u,r)$, then $E_r(u)$ is the enclosure region $E(u)$ of u.*

PROOF. Consider any node w that is not inside $B(u,r)$. For any point x in the relay region $R(u,w)$, we have $||ux|| > ||uw|| > r$. It then implies that the intersection of $R(u,w)$ and $E_r(u)$ is empty. Then node w can not affect the enclosure region $E(u)$. ◻

Then the total power used by the above approach is $\sum_{j=0}^{i-1} 2^j p = (2^i - 1)p$. And the optimum protocol will use the power between $2^{i-2}p$ and $2^{i-1}p$, assuming that the protocol can guess correctly the optimum transmission range. Then the power consumed by our protocol is within $\frac{(2^i-1)p}{2^{i-2}p} < 4$ times of the optimum power consumed. Then we have the following theorem.

Theorem 12 *The power consumed by the above doubling approach to find the region that contains all information necessary for computing the neighbors of a node u is less than 4 times of the optimal power consumption.*

Notice that we did not consider the receiver's power cost in above proof. For example, for a node v located within the transmission power range $2^j p$ but not within the transmission power range $2^{j-1}p$, it receives the signal from node u $\log \frac{2^{i-1}p}{2^{j-1}p} = i-j$ times. In other words, the receiving cost of that node v is $(i-j) \cdot c$ if node v receives the signal whenever u sends signal. Notice that if there are lots of mobile nodes located within the transmission range $2^i p$ but not $2^{i-1}p$, then the receiving cost could be very large compared to the optimum. However, if the mobile nodes are well-spaced [22], it is not difficult to show that the total receiving cost of all nodes at $T(u)$ is within a small constant factor of the optimum cost.

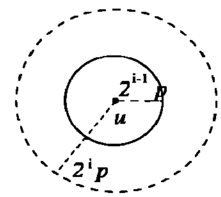

Figure 10: Finding the search region.

V. Dynamic Distributed Networks

For mobile wireless networks, since every node often moves over the time, the networking protocol must be able to dynamically update its links in order to maintain the strong connectivity of the network. All our discussions had been concentrated on the stationary network, in which all nodes are assumed to be static and work perfectly. In other words, no new node is added into the network, nor any node will go out of work. In this section, we consider the case that the network is dynamically changing. Notice that a node moves from one position to the other position can be viewed as two events: one node is deactivated at the old position and one node is activated at the new position. Therefore, in the following, we only consider how to add a new node to the network and how to remove one node from the network.

First let's consider how to add a new node into the network. Notice that, for each node u, we only use the nodes from $N(u)$ to relay the signal sent from u if necessary. Assume that node z is added to the network. It is easy to show that only some node u whose enclosure region $E(u)$ contains z need to be updated. To update the networking topology, the new node z broadcasts its position information to nearby nodes. Each node u that received the message checks whether the node z is contained in its enclosure regions. Assume that node u also stores a set of nodes that defines the enclosure region $E(u)$. Node u checks if there is a node v defining $E(u)$ such that z is inside $R(u,v)$. If such node v exists, then the neighbor set $N(u)$ does not need to be updated. Otherwise, remove all nodes $v \in N(u)$ such that v is in $R(u,z)$. It is easy to show that the above procedure can be done in $O(N(u))$ time. Obviously, we can update the enclosure region $E(u)$ in $O(|E(u)|)$ time, where $|E(u)|$ is the number of nodes defining it.

Then we consider how to remove a node from the network. Obviously, a removed node z affects the neighbors of a node u when $z \in N(u)$. However, when node z defines a curve in $E(u)$, then the removal of node z will affect the enclosure region $E(u)$. Consequently, it may introduce some new neighbors to the node u. Therefore, we first check if z belongs to the set of nodes defining $E(u)$. If it does, we have to revive the nodes blocked by z only and add them to $N(u)$. The set of nodes defining $E(u)$ is also updated correspondingly. The above procedure can be done in $O(d_{G_t}(u)\delta(u))$ time, where $\delta(u)$ is the number of new neighbors introduced. The updating of the new neighbors is similar to finding all neighbors.

The main advantage of our algorithm over the algorithm proposed in [18] is as following. The algorithm proposed in [18] tries to recompute all neighbors information after a node wakes up. It is too expensive to do so. On the other hand, our algorithm tries to use as much previous information as possible. The node only needs to update the neighbors information only if some nearby nodes moved during its sleep time.

VI. Conclusion

We have described a distributed protocol to find an enclosure graph that approximates the minimum-power topology for a stationary wireless ad hoc network. Assume a node u has found $d_{G_t}(u)$ nodes within its transmission range. We proposed a $O(\min(d_{G_t}(u)d_{G_e}(u), d_{G_t}(u)\log d_{G_t}(u)))$ time complexity algorithm to compute the neighbors of u, where $d_{G_e}(u)$ is the number of neighbors of node u. We also show how to updating the topology when the network is dynamically changing.

After the minimum-power topology is constructed, the Bellman-ford algorithm can then be applied to compute the shortest path between any two nodes. However, the distributed Bellman-ford algorithm may be too slow to compute to the shortest path. It is worthwhile to develope an algorithm which can directly find the shortest path or find the path whose length is within a constant factor of the shortest path. Here the *length* of a path is the energy consumed by this path. The routing must be truly local. In other words, it only uses the destination location information and the current node information such as its location and its neighbors.

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