Localized Routing for Wireless Ad Hoc Networks

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***Abstract*— We show that, given a set of randomly distributed wireless nodes with density n, when the transmission range r_n of wireless nodes satis£es $\pi r_n^2 \geq 3\frac{\log n + c(n)}{n}$, the localized Delaunay triangulation (LDel) [1] is the same as the Delaunay triangulation with high probability, where $c(n) \to \infty$ as n goes in£nity. Our experiments show that the delivery rates of existing localized routing protocols are increased when localized Delaunay triangulation is used instead of several previously proposed topologies, and the localized routing protocol based on Delaunay triangulation works well in practice.**

I. Introduction

One of the key challenges in the design of *ad hoc* networks is the development of dynamic routing protocols to £nd routes ef£ciently between two communication nodes. In recent years, a variety of routing protocols [2], [3], [4], [5], [6], [7] targeted speci£cally for *ad hoc* environment were developed.

Several researchers proposed another set of routing protocols, namely the localized routing, which select the next node to forward the packets based on the position of its local neighbors and the destination node. Most of the protocols [8], [9], [10] use some *planar* network topologies, e.g., the Gabriel graph [11], that can be constructed ef£ciently in a distributed manner. Using Gabriel graph although can guarantee the delivery of the packets with the help of the right-hand rule when simple greedy-based routing heuristics fail, however, the distance traveled by the packet could be much larger than the minimum required [12], [13], [14], i.e., Gabriel graph is not a good approximation of the unit disk graph in terms of the pair-wise distance between wireless nodes. Formally, given a graph H, a spanning subgraph G of H is a t-spanner if the length of the shortest path connecting any two nodes in G is no more than t times the length of the shortest path connecting them in H. Li *et al.* [1] designed a localized algorithm to construct a planar t-spanner for the unit-disk graph such that some localized routing protocols can be applied on it. They called the constructed graph planarized *local Delaunay triangulation*, denoted by $PLDel$.

Applying localized routing methods [9], [10] on $PLDel$, a better performance is expected because $PLDel$ is denser compared to the Gabriel graph, but still with $O(n)$ edges. However, these two methods do not guarantee that the ratio between the distance traveled by the packets to the minimum possible. The method proposed by Bose and Morrin [15] does guarantee this distance ratio, but that needs the construction of the Delaunay triangulation, which cannot be constructed and updated ef£ciently in a distributed manner and routing based on it might not be possible since it can contain links longer than the transmission radius of nodes.

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We prove that the localized Delaunay triangulation almost surely contains the Delaunay triangulation of the set n of randomly distributed wireless nodes when the transmission range r_n satis£es $\pi r_n^2 \geq 3\frac{\ln n + c(n)}{n}$, where $c(n) \to \infty$ as n goes to in£nity. Notice that, Gupta and Kumar [16] showed that the unit disk graph is connected with high probability if the transmission range r_n satis£es $\pi \cdot r_n^2 \geq \frac{\ln n + c(n)}{n}$ for any $c(n)$ with $c(n) \to \infty$ as n goes in£nity. Thus, with high probability, we can construct the Delaunay triangulation $Del(V)$ by constructing the local Delaunay triangulation instead. We then present a localized routing method that guarantees that the distance traveled by the packet is no more than a small constant factor of the minimum using the property of Delaunay triangulation.

We study the performance of our localized routing method by some simulations in which results show the delivery is guaranteed and the ratio of the length traveled by packet to the minimum is small. Our simulations show that the delivery rates of several localized routing protocols are also increased when the localized Delaunay triangulation is used. In our experiments, several simple local routing heuristics, applied on the localized Delaunay triangulation, have always successfully delivered the packets, while other heuristics were successful in over 90% of the random instances. Moreover, because the constructed topology is planar, a localized routing algorithm using the right hand rule guarantees the delivery of the packets when simple heuristics fail. The experiments also show that several localized routing algorithms (notably, compass routing [17] and greedy routing) also result in a path whose length is within a small constant factor of the shortest path; we already know such a path exists since the localized Delaunay triangulation is a t-spanner.

The remaining of the paper is organized as follows. In Section II, we review some de£nitions, structures, and the prior art on the localized routing methods. In Section III, we show that the Delaunay triangulation can be constructed through the local Delaunay triangulation with high probability. We then present the fully localized routing algorithm that guarantees a small spanning ratio in Section IV. We study the performance of various localized routing algorithms in Section V. Section VI gives a brief conclusion of our paper.

II. Preliminaries

Assume that all wireless nodes are given as a set V of n nodes in a two dimensional space. Each node has some computational power and a distinctive identity. Additionally, each static wireless node knows its position information, either through a low-power GPS receiver or through some other way. Let $x(v)$ and $y(v)$ be the value of the x-coordinate and y-coordinate of a node v respectively. Our results actually only requires each node knows the relative positions of its neighbors. For simplicity, all wireless nodes have the same maximum transmission range, denoted by r_n. By a simple broadcasting, each node u can gather the location information of all nodes within its transmission range. Consequently, all wireless nodes V together de£ne a unit-disk graph (UDG) $G(V, r_n)$, which has an edge uv iff the Euclidean distance $\|uv\|$ between u and v is less than r_n. We normalize r_n to one unit if no confusion is caused.

A. Proximity Graphs

Various proximity graphs [18], e.g., relative neighborhood graph (RNG) [19], Gabriel graph (GG) [19], and Yao graph (YG) [20], were de£ned for UDG. Although Yao graph has constant spanning ratio [21], [22], [18] it is not planar. The relative neighborhood graph and the Gabriel graph are planar graphs, but they are not a spanner for the unit-disk graph [12].

Assume that no four nodes of V are co-circular. A triangulation of V is a *Delaunay triangulation*, denoted by $Del(V)$, if the circumcircle of each of its triangles does not contain any node of V in its interior. The *Voronoi region* of a node p in V, denoted by $Vor(p)$, is the collection of two dimensional points which is closer to p than to any other node of V. The *Voronoi diagram* for V is the union of all Voronoi regions $Vor(p)$, where $p \in V$. The shared boundary of two Voronoi regions $Vor(p)$, $Vor(q)$ is on the perpendicular bisector line of segment pq. The boundary segment of a Voronoi region is called the *Voronoi edge*. The intersection point of two Voronoi edge is called the *Voronoi vertex*. It is well-known [23], [24], [25] that $Del(V)$ is a planar t-spanner of the completed Euclidean graph $K(V)$.

B. Local Delaunay Triangulation

Given a set of points V, let $UDel(V)$, the *unit Delaunay triangulation*, be the graph obtained by removing all edges of $Del(V)$ that are longer than r_n. It is easy to show that nodes u, v and w together can not decide if they can form a triangle $\triangle uvw$ in $Del(V)$ by using only their local information. The following de£nition was given by Li *et al.* [1].

De£nition 1: A triangle $\triangle uvw$ is *k-localized Delaunay* if (1) the interior of circumcircle $disk(u, v, w)$ does not contain any node of V that is a k-neighbor of u, v, or w; (2) all edges of the triangle $\triangle uvw$ have length no more than one unit.

De£nition 2: The *k-localized Delaunay graph* over a node set V, denoted by $LDel^{(k)}(V)$, has exactly Gabriel edges and edges of k-localized Delaunay triangles.

Li *et al.* [1] proved that $UDel(V) \subseteq LDel^{(k)}(V) \subseteq LDel^{(k+1)}(V)$. It implies each edge uv of $UDel(V)$ is either a Gabriel edge or forms a 1-localized Delaunay triangle with some edges from $UDel(V)$. They also proved that $LDel^{(k)}(V)$ is a planar graph for $k \geq 2$. but $LDel^{(1)}(V)$ is not always. They gave ef£cient method to remove the intersections in $LDel^{(1)}(V)$ and called the resulting graph Planarized Local Delaunay Triangulation (PLDel), which contains $UDel$ as a subgraph. Thus, if the longest edge of $Del(V)$ is at most one unit, obviously, $PLDel$ is the Delaunay triangulation.

C. Localized Routing Algorithms

Let $N_k(u)$ be the set of nodes of V that are within k hops distance of u in the unit-disk graph $UDG(V)$. After one successful transmission by every node, each node u of V knows the location and identity information of all nodes in $N_1(u)$.

Assume a packet is currently at node u, and the destination node is t. Several localized routing algorithms, i.e., £nd the next node v of u based on t and $N_k(u)$, were developed.

- COMPASS ROUTING(CMP): The relay node v forms the smallest angle $\angle vut$ among all neighbors of u. See[17].
- RANDOM COMPASS ROUTING(RCMP): Let v_1 be the node above line ut such that $\angle v_1ut$ is the smallest among all such neighbors of u. Similarly, let v_2 be node below line ut that minimizes the angle $\angle v_2ut$. Then node u randomly choose v_1 or v_2 to forward the packet. See[17].
- GREEDY ROUTING(GRDY): Node u £nds neighbor v closest to t as relay node. See [9].
- MOST FORWARDING ROUTING (MFR): Node u £nds neighbor v such that $\|v't\|$ is the smallest as relay node, where v' is the projection of v on segment ut. See [8].
- NEAREST NEIGHBOR ROUTING (NN): Given a parameter angle α, node u £nds the nearest node v as forwarding node among all neighbors of u such that $\angle vut \leq \alpha$.
- FARTHEST NEIGHBOR ROUTING (FN): Given a parameter angle α, node u £nds the farthest node v as forwarding node among all neighbors of u such that $\angle vut \leq \alpha$.
- GREEDY-COMPASS(GCMP): Node u £nds the neighbors v_1 and v_2 that forms the smallest clockwise and counter-clockwise angle respectively among all $N_1(u)$ with the segment ut. The packet is forwarded to the node of $\{v_1, v_2\}$ with the minimum distance to t. See [15], [26]

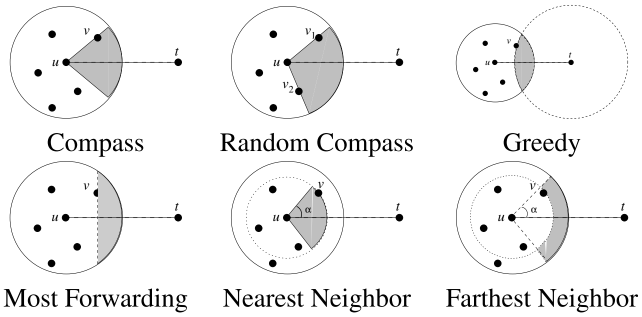

Fig. 1. Various localized routing methods. Shaded area is empty of nodes.

The compass routing, random compass routing and the greedy routing guarantee to deliver the packets if Del is used as network topology [9], [17]. Morin [26] proved that: (1) the greedy routing guarantees the delivery of the packets if the underlying structure is Del; (2) the compass routing guarantees the delivery of the packets if the regular triangulation is used as the underlying structure. (3) the greedy-compass routing works for all triangulations, i.e., it guarantees the delivery of the packets as long as a triangulation is used as the underlying structure. Notice Del is a special regular triangulation. There are triangulations (not Delaunay) that defeat greedy and compass schemes.

In order to make the localized routing work, the source node has to learn the current (or approximately current) location of the destination node. Notice that, for sensor networks collecting data, the destination node is often £xed, thus, location service is not needed in these applications. However, the help of a *location service* is needed in most application scenarios. For the state of art of location service, see survey by Mauve *et al.*[27].

III. Construct Delaunay Locally

Several researchers [16], [28] proved that the random point process bears the same stochastic property as the homogeneous Poisson point process, in which the probability of exactly k nodes appearing in a region Ψ of area A is $\frac{(nA)^k}{k!} \cdot e^{-nA}$, where n is the density of the Poisson process. We consider the homogeneous Poisson point process instead of uniform process.

Let D_n be random variable for the length of the longest edge of $Del(V)$ for V generated by a homogeneous Poisson process with density n. The circumcircle of a Delaunay triangle $\triangle pqs$ has area at least $\pi d^2/3$ if the longest edge $\|pq\| \leq d$. The probability, denoted by p_1, that this circumcircle is empty of nodes is at most $e^{-n\pi d^2/3}$. Since there are at most $3n$ triangles in the two-dimensional Delaunay triangulation of n nodes, $Pr(D_n \leq d) \leq 3n{\cdot}p_1 \leq 3n{\cdot}e^{-n\pi d^2/3}$. By solving the inequality $3n \cdot e^{-n\pi d^2/3} \leq \frac{1}{\beta}$, we know that, with probability at most $\frac{1}{\beta}$, $D_n \geq d$, where $n\pi d^2 = 3(\ln n + \ln \beta + \ln 3)$. Thus, with probability at least $1 - \frac{1}{\beta}$, $D_n \leq \sqrt{3(\ln n + \ln \beta + \ln 3)/(n\pi)}$.

Penrose [29] showed that the longest edge of the minimum spanning tree of homogeneous Poisson point process $\mathcal{P}_n$ is at most M_n with probability $e^{-e^{-\alpha}}$, where $n\pi M_n^2 \leq \ln n + \alpha$. In other words, if the transmission radius r_n satis£es $n\pi r_n^2 \geq \ln n + \alpha$, then $G(V, r_n)$ is connected with probability at least $e^{-e^{-\alpha}}$ as n goes to in£nity. By substituting $e^{\alpha} = \gamma$, with probability at least $1-\frac{1}{\gamma}$, $G(V, r_n)$ is connected if $n\pi r_n^2 \geq \ln n + \ln \gamma$.

Consequently, $G(V, r_n)$ is connected with probability at least $1 - \frac{1}{n^5}$ if $n\pi r_n^2 \geq 6 \ln n$; meanwhile, with probability at least $1 - \frac{1}{n}$, the longest edge of $Del(V)$ is at most r_n. To make $G(V, r_n)$ connected with probability $1 - \frac{1}{n}$, we need $n\pi r_n^2 \geq 2 \ln n$, i.e., the required transmission range so that $PLDel$ equals Del is just $\sqrt{3/2}$ of the minimum transmission range to have a connected network with high probability.

Previously, we did not consider the boundary effects: when Ω is bounded, $disk(p, q, s)$ of a Delaunay triangle $\triangle pqs$ near the domain boundary is not fully contained in Ω. Thus, the probability that $disk(p, q, s)$ is empty of other nodes, which depends on the area of $disk(p, q, s) \cap \Omega$, is larger than the unbounded case, i.e., long edges more likely appear near the domain boundary. When Ω is a disk with unit radius, the area of $disk(p, q, s) \cap \Omega$ is at least $d^3/4$ if $\|pq\| = d$. Similarly, the longest edge length of $Del(V)$ satis£es a transition phenomena: with probability $\geq 1 - \frac{1}{\beta}$, $D_n \leq \sqrt[3]{4(\ln n + \ln \beta + \ln 3)/n\pi}$. The proof is omitted here due to space limit.

IV. Routing Based on Delaunay

Bose and Morrin [15] proposed a method to route the packets using $Del(V)$ based on the proof [23] that $Del(V)$ is a spanner. Dobkin *et al,* [23] construct a path $\Pi_{dfs}(u, v)$ in $Del(V)$ with length no more $(1 + \sqrt{5})\pi\|uv\|/2$, which consists of two parts: one is *direct DT* subpaths, the other is *shortcut* subpaths.

De£ne the *tunnel*, denoted by $T(u, v)$, of segment uv as the set of triangles in $Del(V)$, whose interior intersects the segment uv. The triangles illustrated in Figure 2 is the tunnel $T(u, v)$.

Given two nodes u and v, let $b_0 = u$, b_1, b_2, $\cdots$, b_{m-1}, $b_m = v$ be the nodes corresponding to the sequence of Voronoi regions traversed by walking from u to v along the segment uv. See Figure 2 for an illustration. If a Voronoi edge or a Voronoi vertex happens to lie on the segment uv, then choose the Voronoi region lying above uv. Assume that the line uv is the x-axis. The sequence of nodes b_i, $0 \leq i \leq m$, de£nes a path from u to v, which is the *direct DT path* from u to v [23].

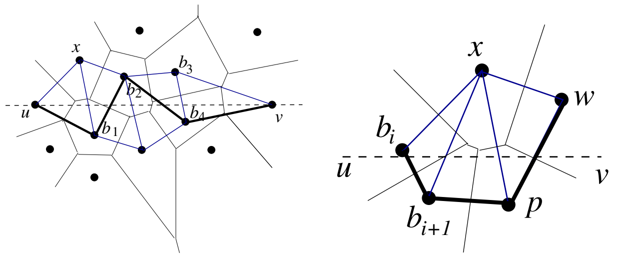

Fig. 2. Left: The direct DT path $ub_1b_2b_3b_4v$ is shown by thickest lines. The thin lines represent the Voronoi diagram. Right: Find the next neighbor of node b_i in the direct DT path or the neighbor of x in the shortcut path.

The path $\Pi_{dfs}(u, v)$ uses the direct DT path as long as it is above the x-axis. Assume that the path constructed so far has brought us to some node b_i such that $y(b_i) \geq 0$, $b_i \neq v$, and $y(b_{i+1}) < 0$. Let j be the least integer larger than i such that $y(b_j) \geq 0$. Then the constructed path uses either the direct DT path from b_i to b_j or takes a *shortcut*, which is the upper boundary of $T(u, v)$ that connects b_i and b_j. Let x_i, x_j be the x-coordinates of b_i and b_j respectively. Let $c_{dfs} = (1 + \sqrt{5})\pi/2$. Then either the length of the direct DT path or the length of the shortcut path from b_i to b_j is at most $c_{dfs}(x_j - x_i)$ [23]; but either path could be arbitrarily larger than the minimum [15].

The routing strategy by Bose and Morin try to follow packets on path $\Pi_{dfs}(u, v)$. The dif£culty occurs as the strategy does not know prior which of the two subpaths is shorter. when the direct DT path leads us to an edge b_ib_{i+1} that intersects uv, Their solution is to simulate exploring both subpaths in a parallel manner whenever the £rst one reaches node b_j. Many technique details need to be £lled so it can be implemented.

For simplicity, let $v_1 = u, v_2, \cdots, v_{k-1}, v_k = v$ be the k vertices of all b_i's that is on or above the segment uv.

Firstly, we study how to £nd the neighbor of a node b_i in the direct DT path locally. Node b_i can compute $Vor(b_i)$ locally since it knows all Delaunay edges incident on b_i and the Voronoi diagram is a dual of the Delaunay triangulation. Then node b_{i+1} is the node that (1) shares the Voronoi edge of $Vor(b_i)$ that is intersected by uv, and (2) has larger x-coordinate than node b_i. See Figure 2.

Secondly, we show how to £nd the next neighbor of a node x in the shortcut path locally. It sorts all incident Delaunay edges in count-clockwise order and £nds the incident neighbor w such that xw does not intersect the segment uv, but the previous incident Delaunay edge intersects uv. See Figure 2.

Thirdly, we reach the node b_j if (1) the Voronoi diagram of the current node intersects uv, (2) it is above uv, and (3) if we are exploring the shortcut path, the Voronoi Diagram of the previous visited node does not intersect uv; if we are exploring the direct shortcut path, the previous visited node is below uv. In Figure 2, node w will be that node b_j.

The routing algorithm works as following. Let $v_0 = u, i = 0$. Let v_{i+1} be the node returned by Explore(v_i). If v_{i+1} is not destination, increase i by one and call Explore(v_i).

Algorithm: EXPLORE(v_i)

Let p_0, q_0 be the next neighbor of v_i in the direct DT path, and the shortcut path respectively. Let $j = 0$ and $l_0 = \min(\|v_i p_0\|, \|v_i q_0\|)$. Repeat the following exploring of both direct DT and the shortcut path until a node, denoted by v_{i+1}, which is on the direct DT path and is above uv, is reached. If $\|v_i p_0\| \le \|v_i q_0\|$, we explore the direct DT path £rst. Otherwise, we explore the shortcut path £rst.

EXPLORE DIRECT DT PATH: Route the packet along the direct DT path from node v_i until reaching node v_{i+1} or reaching a node, say p_{j+1}, such that the distance traveled from v_i to p_{j+1} is larger than $2l_j$ for the £rst time. If node v_{i+1} is reached, return v_{i+1} and quit. Otherwise, set $j = j + 1$ and l_j be the distance traveled from v_i to p_{j+1}, and then return to node v_i.

EXPLORE SHORTCUT PATH: Route the packet along the shortcut path from node v_i until reaching node v_{i+1} or reaching a node, say q_{j+1}, such that the distance traveled from v_i to q_{j+1} is larger than $2l_j$ for the £rst time. If node v_{i+1} is reached, return v_{i+1} and quit. Otherwise, set $j = j + 1$ and l_j be the distance traveled from v_i to q_{j+1}, and then return to node v_i.

If we always start exploring the shortcut path £rst, this may lead to a long traveling distance. The distance traveled by the above routing strategy is $9c_{dfs}$-competitive [26].

V. EXPERIMENTS

We £rst study the transition phenomena of the longest edge of the Delaunay triangulation. In our experiments, three different geometry regions Ω: disk with radius $200m$, square with side $400m$, and unbounded (with unit $400m$), are tested. The node density n is 50, 100, 200, 300, 400, and 500. For each choice of Ω and n, 10000 sample of n points is generated, and the longest Delaunay edge is generated for each sample. Left £gures of Figure 3 illustrate the longest Delaunay edge length D_n distribution, while the right £gures illustrate its transition phenomena. The statistics for D_n is from 0 to 400 meters, using 4 meters increment. Interestingly, for square region, varying density n does not change the distribution and transition at all statistically. The transition in the circular region is slower than the counterpart in the unbounded region. We found $D_n \le 130m$ almost surely for circular region with $n = 100$.

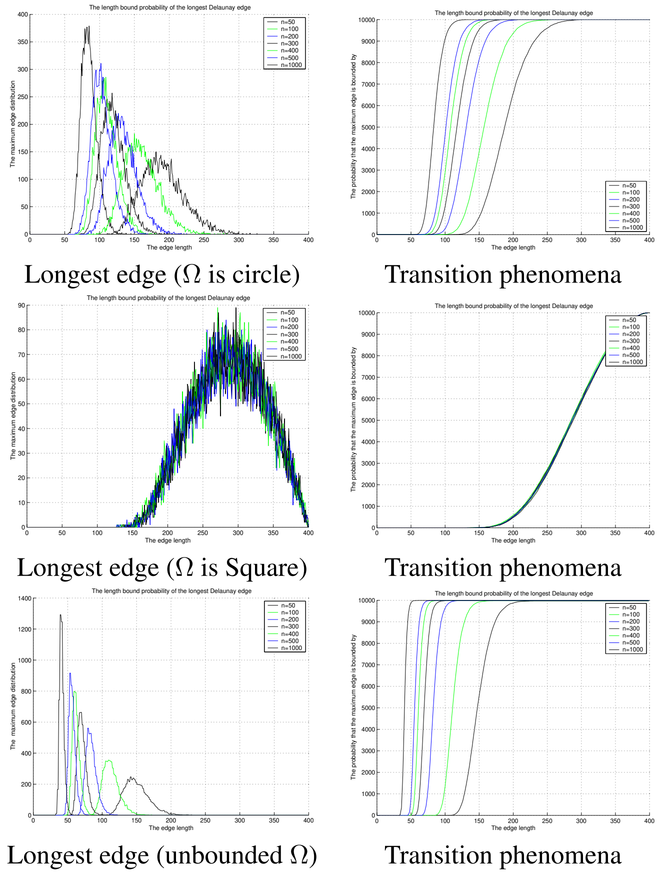

Fig. 3. Transition phenomena of D_n when Ω is circle, square, and unbounded.

We then present our experiments of various routing methods on different topologies. We choose 100 nodes distributed randomly in a circular area with radius 100 meters. Each node is speci£ed by a random x, y coordinate, with transmission radius 30 meters. Figure 4 illustrates some discussed topologies. We randomly select 20% of nodes as source; and for each source, we randomly choose 20% of nodes as destination. The statistics are computed over 10 different node sets. We found that $LDel^{(2)}(V)$ and $PLDel(V)$ are almost the same as $Del(V)$. The differences lye near the boundary. These two Graphs are preferred over the Yao graph because we can apply the right hand rule when the simple heuristic localized routing fails.

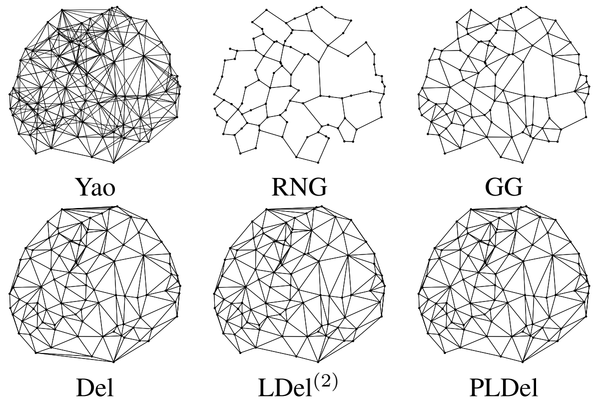

Fig. 4. Various planar network topologies (except Yao).

Table I illustrates the delivery rates. For routing methods NN and FN, we choose the next node within $\pi/3$ of the destination direction. Because $LDel^{(2)}(V)$ and $PLDel(V)$ are much denser than all previous known planar topologies such as GG and RNG, the delivery rates of many routing methods on them are near or equal 100%. Interestingly, we found that when Yao graph is used, the delivery rates are high in all methods. The reason these methods delivered the packets when Yao structure is used could be: there is a node within the transmission range in the direction of the destination with high probability when $N_1(u)$ is large enough. Although it was proved [26] that methods Cmp, RCmp, Grdy, and RGrdy guarantees the delivery if $Del(V)$ is used, we found that only compass, greedy, and greedy-compass methods did in our simulations. Because we set the hop limit of all methods as n and the expected number of visited nodes by random compass could be $O(n^2)$ [26], it is not suprise that random compass did not deliver all packets. Table

II illustrates the maximum spanning ratios of the path traversed by the packet from source s to destination t to $\|st\|$. We are investigating the theoretical reason why the spanning ratios of compass and random compass methods are so large. Although the maximum spanning ratio by DTR is larger than most previous methods, DTR is the only known method that guarantees this spanning ratio to be a constant.

TABLE I
THE DELIVERY RATE.

	Yao	RNG	GG	Del	LDel$^{(2)}$	PLDel
NN	100	20.4	83.3	100	100	98.4
FN	94.5	25.8	70.2	94.6	100	95.2
MFR	98.6	54.5	90	97.4	95	97.2
Cmp	97.1	23.2	66.2	100	100	100
RCmp	95.4	51.4	66.2	85.7	86.9	89.9
Grdy	100	78.3	100	100	100	100
GCmp	95	26.5	76.6	100	98.4	100
DTR				100	100	100

TABLE II
THE MAXIMUM SPANNING RATIO.

	Yao	RNG	GG	Del	LDel$^{(2)}$	PLDel
NN	1.7	1.3	1.6	1.5	1.6	1.6
FN	3.3	1.6	1.8	1.9	2.3	2.4
MFR	4.7	1.7	2.3	1.8	1.8	3.1
Cmp	11	2.2	6.2	16	18	18
RCmp	27	20	19	31	27	21
Grdy	1.7	2.0	1.8	1.6	1.5	1.6
GCmp	2.5	1.6	2.7	1.9	1.9	1.9
DTR				8.6	8.6	8.4

VI. CONCLUSION

We showed that, given a set of randomly distributed wireless nodes with density n, when $\pi r_n^2 \geq \frac{6 \log n}{n}$, $PLDel$ equals Del with probability $\geq 1 - \frac{1}{n}$. If $\pi r_n^2 \geq \frac{6 \log n}{n}$, the network $G(V, r_n)$ is connected with probability $\geq 1 - \frac{1}{n^5}$. In other words, we can construct Del using $PLDel$ with high probability. We then discussed a localized routing protocol that guarantees the distance traveled by the packets is no more than a constant factor of the minimum. We also conducted experiments to study (1) the transition phenomena of the longest Delaunay edge length and (2) the delivery rates and the spanning ratio of existing localized routing protocols when localized Delaunay triangulation and several previous known structures are used.

Notice that the Delaunay based routing method DTR works only when a Delaunay triangulation is obtained. We leave it as a future work to design a protocol that can guarantee packet delivery and the traveled distance using $PLDel$.

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