A Unified Energy-Efficient Topology for Unicast and Broadcast

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ABSTRACT

We propose a novel communication efficient topology control algorithm for each wireless node to select communication neighbors and adjust its transmission power, such that all nodes together self-form a topology that is energy efficient simultaneously for both unicast and broadcast communications. We prove that the proposed topology is *planar*, which guarantees packet delivery if a certain localized routing method is used; it is power efficient for unicast– the energy needed to connect any pair of nodes is within a small constant factor of the minimum under a common power attenuation model; it is efficient for broadcast: the energy consumption for broadcasting data on top of it is asymptotically the best compared with structures constructed locally; it has a constant bounded logical degree, which will potentially reduce interference and signal contention. We further prove that the average physical degree of all nodes is bounded by a small constant. To the best of our knowledge, this is the *first* communication-efficient distributed algorithm to achieve all these properties. Previously, only a centralized algorithm was reported in [3]. Moreover, by assuming that the ID and position of every node can be represented in $O(\log n)$ bits for a wireless network of n nodes, our method uses at most $13n$ messages, where each message is of $O(\log n)$ bits. We also show that this structure can be efficiently updated for dynamical network environment. Our theoretical results are corroborated in the simulations.

Categories and Subject Descriptors

C.2.1 [**Network Architecture and Design**]: Wireless communication, Network topology; G.2.2 [**Graph Theory**]: Network problems, Graph algorithms

General Terms

Algorithms, Design, Theory

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Keywords

Graph theory, wireless ad hoc networks, topology control, power efficient, low weight, interference, unicast, broadcast.

1. INTRODUCTION

A wireless *ad hoc* network consists of a distribution of radios in a certain geographical area. Unlike cellular wireless networks, there is no centralized control in the network, and wireless devices (called *nodes* hereafter) can communicate via multi-hop wireless channels: a node can reach all nodes inside its transmission range while two far-away nodes communicate through the relaying by intermediate nodes. An important requirement of these networks is that they should be self-organizing, *i.e.*, transmission ranges and data paths are dynamically restructured with changing topology. Energy conservation and network performance are probably the most critical issues in wireless *ad hoc* (and sensor) networks, because wireless devices are usually powered by batteries only and have limited computing capability and memory.

A wireless *ad hoc* or sensor network is modelled by a set V of n wireless nodes distributed in a two-dimensional plane. Each node has the same *maximum* transmission range R meters, *e.g.*, a typical 802.11g wireless LAN adapter has a transmission range around $100m - 500m$. By a proper scaling, we assume that all nodes have the maximum transmission range equal to one unit. These wireless nodes define a *unit disk graph* $UDG(V)$ in which there is an edge between two nodes iff the Euclidean distance between them is at most one unit. In other words, we assume that two nodes can always receive the signal from each other directly if the Euclidean distance between them is no more than one unit. Notice that, in practice, the transmission region of a node is not necessarily a perfect *disk*. As done by most results in the literature, for simplicity, we model it by *disk* in order to first explore the underlying nature of ad hoc networks. Hereafter, $UDG(V)$ is always assumed to be connected. We also assume that all wireless nodes have distinctive identities(IDs). Additionally we assume that each node knows the relative position of its one-hop neighbors. The relative position of neighbors can be estimated by the *direction of signal arrival* and the *strength of signal*. The geometry location of a wireless node can also be obtained by a localization method, such as [35, 8, 15]. We here assume that the localization is low cost or it is already required by some other protocols. Notice that the higher these localization costs, the less desirable the advocated approach to design any protocol based on nodes' geometry location.

We adopt the most common power-attenuation model from literature: the power needed to support a link uv is assumed to be $\|uv\|^{\beta}$, where $\|uv\|$ is the Euclidean distance between u and v, β is a real constant between 2 and 5 depending on the wireless transmission environment. Throughout this paper we only focus on the transmission power of all nodes. This energy model only accounts for the emission power. This can be a good approximation of what happens in case of long range techniques although the actual energy consumption is given by a fixed part (receiving power and the power needed to keep the electric circuits on) plus the emission power component. In other words, we assume that the transmission range is large enough such that the emission power is the major component and the receiving power is negligible. Notice that, as pointed out by an anonymous reviewer, even if the energy cost of receiving a packet is high, there are a number of ways of reducing this cost by reducing the number of packets received by but not intended for a node. It includes, but is not limited to, the following approaches: (1) "*signals are sent with special small preambles that identify the intended recipient*"; (2) "*the radios are frequency-agile and can choose different frequency channels to communicate with different neighbors*" (3) "*the radios have directional antennas which limit the volume over which their signals are received*"; (4) "*favoring routes that traverse sparser portions of the network*".

The (localized) *topology control* technique lets each wireless device (*locally*) adjust its transmission range and select certain neighbors for communication, while maintaining a decent global structure to support energy efficient routing and to improve the overall network performance. A distributed method is *localized* if it runs in constant number of rounds [34]. By enabling each wireless node to shrink its transmission power (which is usually much smaller than its maximum transmission power) sufficiently enough to cover its farthest selected neighbor, topology control schemes can not only save energy and prolong network life, but also improve the network throughput through mitigating the MAC-level medium contention. Unlike traditional wired and cellular networks, the movement of wireless devices during the communication could change the network topology to some extent. Hence, it is more challenging to design a topology control algorithm for *ad hoc* wireless networks: the topology should be locally and self-adaptively maintained using low communication cost, without affecting the whole network's performance.

The main contributions of this paper are as follows. We present the *first* communication efficient algorithm to construct a *unified* energy-efficient topology for unicast and broadcast in wireless ad hoc/sensor networks. In one single structure, we guarantee the following network properties:

1. **power efficient unicast**: given any two nodes, there is a path connecting them in the structure with total power cost no more than $2\rho+1$ times of the power cost of any path connecting them in the original network. Here $\rho > 1$ is some constant that will be specified later.
2. **power efficient broadcast**: the power consumption for broadcast is within a constant factor of optimum among all *locally* constructed structures. Notice that our structure cannot guarantee that the energy consumption is within a constant factor of the *global* optimum. Essentially, we prove that the structure is *low-weighted*: its total edge length is within a constant factor of that of Euclidean Minimum Spanning Tree.
3. **bounded logical node degree**: each node has to communicate with at most $k-1$ logical neighbors, where $k \geq 9$ is an adjustable parameter;
4. **bounded average physical node degree**: the average physical node degree is bounded from above by a small constant. Here the physical degree of a node u in a structure H is defined as the number of nodes inside the disk centered at u with radius $\max_{uv \in H} \|uv\|$.
5. **planar**: there are no edges crossing each other. This enables several localized routing algorithms, such as [2, 18, 24, 25], to be performed on top of this structure to guarantee the packet delivery without routing table.
6. **neighbors Θ-separated**: the directions between any two logical neighbors of any node are separated by at least an angle θ, which as we will see reduces the signal interference. It also can be used to reduce the receiving power cost when directional antenna is used.

In graph theoretical terminologies, given a unit disk graph modelling the wireless ad hoc networks, we propose an communication efficient distributed method to build a low-weighted planar power-spanner with a bounded logical node degree. Here a geometric structure is called *low-weighted* if its total edge length is no more than a small constant factor of that of the Euclidean minimum spanning tree. To the best of our knowledge, it is the *first* known *communication efficient* algorithm to construct such a *single* structure with all these desired properties. Previously, only a centralized algorithm was reported in [3]. Moreover, by assuming that the ID and position of every node can be represented in $O(\log n)$ bits for a wireless network of n nodes, we show that the structure can be initially constructed using at most $5n$ to $13n$ messages.

In addition, we prove that the expected average node interference (which is defined as the number of nodes within its adjusted transmission range) in the structure is bounded by a small constant. This is significant in its own due to the following reasons: it has been taken for granted that "*a network topology with a small logical node degree will guarantee a small interference*" and recently Burkhart *et al.* [5] showed that this is not true generally. Our results show that, although generally a small logical node degree cannot guarantee a small interference, the expected average interference is indeed small if the logical communication neighbors are chosen carefully. All our theoretical results are corroborated in simulations.

We also show that our structure can be easily updated in a dynamic environment when a node moves or dies after the battery power is drained. When a node moves, the topology can be dynamically self-maintained without affecting the whole network, since each node adjusts its transmission range and selects neighbors only according to its neighbor information.

To facilitate the efficient construction of such a unified energy-efficient topology, in the paper, we will first give an improved method to construct degree-bounded planar spanner by using relative positions only. The new structure has the same power spanning ratio $\rho = \frac{\sqrt{2}^{\beta}}{1-(2\sqrt{2}\sin\frac{\pi}{k})^{\beta}}$ as the structure proposed in [42]. Here $k \geq 9$ is a customizable parameter. In addition, the directions between any two neighbors of each node are separated by at least a certain angle θ depending on the parameter k. Simulations show that the node interference in our new structure is indeed smaller than the structure proposed in [42].

The rest of the paper is organized as follows. In Section 2, we review some prior arts in topology control, and summarize some preferred properties of network topology for unicast and broadcast. Section 3 presents an improved algorithm to build a degree-bounded planar spanner with Θ-separation property. We then propose, in Section 4, the first communication efficient algorithm to construct planar spanner with bounded-degree and low weight. We study the expected interference of various structures in Section 5. In Section 6, we briefly study how to dynamically update the structure if nodes are mobile. In Section 7, we conduct extensive simulations to validate our theoretical results. Finally, we conclude our paper in Section 8.

2. CURRENT STATE OF KNOWLEDGE

2.1 Energy-Efficient Unicast Topology

Several structures have been proposed for topology control in wireless ad hoc networks. The *relative neighborhood graph*, denoted by $RNG(V)$ [43], consists of all edges uv such that the intersection of two circles centered at u and v and with radius $\|uv\|$ does not contain any vertex w from the set V. The *Gabriel graph* [13] $GG(V)$ contains an edge uv if and only if $disk(u, v)$ contains no other points of V, where $disk(u, v)$ is the disk with edge uv as a diameter. For convenience, we also denote GG and RNG as the intersection of $GG(V)$ and $RNG(V)$ with $UDG(V)$ respectively. Both GG and RNG are planar, are connected, and contain the *Euclidean minimum spanning tree*$(EMST)$ of V if UDG is connected. RNG is not power efficient for unicast, since the power stretch factor of RNG is $n-1$ in the worst case [48] and not bounded by a constant even for n nodes randomly distributed (our proof is similar to the proof in [4] and omitted here due to space limit). Both RNG and GG are not degree-bounded. The *Yao graph* [51] with an integer parameter $k > 6$, denoted by $\overrightarrow{YG}_k$, is defined as follows. At each node u, any k equally-separated rays originated at u define k cones. In each cone, choose the shortest edge $uv \in UDG(V)$ among all edges emanated from u, if there is any, and add a directed link $\overrightarrow{uv}$. Ties are broken arbitrarily or by ID. The resulting directed graph is called the *Yao graph*. It is well-known that the Yao structure is power efficient for unicast. Several variations [31] of the Yao structure could have bounded logical node degree also. However, all Yao related structures are not planar.

Li *et al.* [29] proposed the Cone Based Topology Control (CBTC) algorithm to first focus on several desirable properties, in particular being an energy spanner with bounded degree. It is basically similar to the Yao structure for topology control. Each node u finds a power $p_{u,\alpha}$ such that in every cone of degree α surrounding u, there is some node that u can reach with power $p_{u,\alpha}$. Here, nevertheless, we assume that there is a node reachable from u by the maximum power in that cone. Notice that the number of cones to be considered in the traditional Yao structure is a constant k. However, unlike the Yao structure, for each node u, the number of cones needed to be considered in the method proposed in [29] is 2Δ, where each neighboring node v could contribute two cones on both side of segment uv. Then the graph G_α contains all edges uv such that u can communicate with v using power $p_{u,\alpha}$. They proved that, if $\alpha \leq \frac{5\pi}{6}$ and the UDG is connected, then graph G_α is a connected graph. On the other hand, if $\alpha > \frac{5\pi}{6}$, they showed that the connectivity of G_α is not guaranteed by giving some counterexample [29]. Unlike the Yao structure, the final topology G_α is not necessarily a bounded degree graph.

Bose *et al.* [3] proposed a centralized method with running time $O(n \log n)$ to build a degree-bounded planar spanner for a two-dimensional point set. It constructs a planar t-spanner with low-weight for a given nodes set V, for $t = (1+\pi) \cdot C_{del} \simeq 10.02$, such that the node degree is bounded from above by 27. Hereafter, we use $C_{del} < 2.6$ to denote the spanning ratio of the Delaunay triangulation [11, 20, 19]. However, a straightforward distributed implementation of this centralized method takes $O(n^2)$ communications in the worst case for a set V of n nodes.

Wang and Li [46] proposed the first efficient distributed algorithm to build a degree-bounded planar spanner BPS for wireless ad hoc networks. Though their method can achieve three desirable features: planar, degree-bounded, and power efficient, the theoretical bound on the node degree of their structure is a large constant. For example, when $\alpha = \pi/6$, the theoretical bound on node degree is 25. In addition, the communication cost of their method can be very high, although it is $O(n)$ theoretically, which is achieved by applying the method in [6] to collect 2-hop neighbors information. The hidden constant is large: it is several hundreds.

Recently, Song *et al.* [42] proposed two methods to construct degree-bounded power spanner, by applying the ordered Yao structures on Gabriel graph. They achieved better performance with much lower communication cost, compared with the method in [46]. One method in [42] only costs $3n$ messages for the construction, and guarantees that there are at most one neighbor node in each of the $k = 9$ equal-sized cones.

Notice that the structures constructed by the methods proposed in [46, 42] are not guaranteed to be low-weighted. Both structures are planar and degree-bounded. The structure constructed in [42] is only a power-spanner, while the structure constructed in [46] is also a length-spanner. Notice that it is known that a length-spanner is always a power spanner [31]. The main contribution of this paper is that we propose the *first* method to construct a single topology that is planar, length-spanner, bounded-degree, and low-weighted.

In summary, for energy efficient unicast routing, the topology is preferred to have following features:

1. Power Spanner: Formally speaking, a subgraph H is called a *power spanner* of a graph G if there is a positive real constant ρ such that for any two nodes, the power consumption of the shortest path in H is at most ρ times of the power consumption of the shortest path in G. Here ρ is called the *power stretch factor* or *spanning ratio*.

2. Degree Bounded: It is also desirable that the logical node degree in the constructed topology is bounded from above by a small constant. A small node degree could reduce the MAC-level contention and interference, also may help to mitigate the well known hidden and exposed terminal problems. In addition, a structure with a small degree will improve the overall network throughout [22]. Bounded degree structures also find applications in Bluetooth wireless networks since a *master* node can have only 7 active slaves.

3. Planar: A network topology is also preferred to be planar (no two edges crossing each other in the graph) to enable some localized routing algorithms work correctly and efficiently, such as *Greedy Face Routing* (GFG) [2], *Greedy Perimeter Stateless Routing* (GPSR) [18], *Adaptive Face Routing*(AFR) [24], and *Greedy Other Adaptive Face Routing* (GOAFR) [25]. Notice that with planar network topology as the underlying routing structure, these localized routing protocols guarantee the message delivery without using a routing table: each intermediate node can decide which logical neighboring node to forward the packet using only local information and the position of the source and the destination.

2.2 Energy-Efficient Broadcast Topology

Broadcast is also a very important operation in wireless ad hoc/sensor networks, as it provides an efficient way of communication that does not require global information and functions well with topology changes. For example, many unicast routing protocols [17, 36, 39, 38, 41] for wireless multi-hop networks use broadcast in the stage of route discovery. Similarly, several information dissemination protocols in wireless sensor networks use some forms of broadcast/multicast for solicitation or collection of sensor information [14, 16, 52]. Since sensor networks mainly [1] use broadcast for communication, how to deliver messages to all the wireless devices in a scalable and power-efficient manner has drawn more and more attention. Not until recently have research efforts been made to devise power-efficient broadcast structures for wireless ad hoc networks.

Notice that, a broadcast routing protocol can be interpreted as *flood-based* broadcasting on a *certain* subgraph of the original communication networks, since *any* broadcast routing can be viewed as an arborescence (a directed tree) T, rooted at the source node of the broadcasting, that spans all nodes. The tree T contains a directed edge $\overrightarrow{uv}$ if node v received the first copy of the broadcast data from node u. Once a broadcast structure H is constructed, the broadcast is a simple flooding on top of H: once a node v got the broadcast message from any of its *logical* neighbors, say u, for the first time, it will simply forward it to all its *logical* neighbors (maybe except the node u) either through one-to-one or one-to-all communications. Let $f_H(p)$ denote the transmission power of the node p required by broadcasting message on top of a broadcast structure H. We assume that the tree H is a directed graph rooted at the source of the broadcasting session: link $\overrightarrow{pq} \in H$ denotes that node p forwarded message to node q. For any leaf node p of H, clearly we have $f_H(p) = 0$ since it does not have to forward the data to any other node. For any internal node p of H, $f_H(p) = \max_{\overrightarrow{pq} \in H} \|pq\|^\beta$ under our energy model if an one-to-all communication model is used; and $f_H(p) = \sum_{\overrightarrow{pq} \in H} \|pq\|^\beta$ under our energy model if an one-to-one communication model is used. The total energy required by H is $\sum_{p \in V} f_H(p)$. In the literature, the one-to-all communication model is typically assumed.

Minimum-energy broadcast routing in a simple ad hoc networking environment has been addressed in [9, 21, 49]. It is known [9] that the minimum-energy broadcast routing problem is NP-hard, *i.e.*, it cannot be solved in polynomial time unless `P=NP`. Three greedy heuristics were proposed in [49] for the minimum-energy broadcast routing problem: EMST (minimum spanning tree), SPT (shortest-path tree), and BIP (broadcasting incremental power). Wan *et al.* [44, 45] showed that the approximation ratios of EMST and BIP are between 6 and 12 and between $\frac{13}{3}$ and 12 respectively; on the other hand, the approximation ratio of SPT is at least $\frac{n}{2}$, where n is the number of nodes. The approximation ratio of EMST is improved to 8 recently by Flammini *et al.* [12]. Unfortunately, none of the above structures can be formed and updated using only linear number of messages, nor locally.

RNG, which can be constructed locally, has been used for broadcasting in wireless ad hoc networks [40]. However, an example was given in [30] to show that the total energy used by broadcasting on RNG could be about $O(n^\beta)$ times of the minimum-energy used by an optimum method. Several practical broadcasting protocols [50, 7, 47] are proposed recently, however, all of them did not provide their theoretical performance bound on the energy consumption. In fact, Li [30] showed that, *no* deterministic localized algorithm can find a structure that approximates the total energy consumption of broadcasting within a constant factor of the optimum. Furthermore, in the worst case, for *any* broadcast based on a locally constructed and connected structure, there is a network configuration of n nodes such that its energy consumption is $\Theta(n^{\beta-1})$ times the optimum. On the other hand, given any low-weighted structure H, *i.e.*, $\omega(H) \leq O(1) \cdot \omega(EMST)$, they proved the following lemma

Lemma 1. *[30]* $\omega_\beta(H) \leq O(n^{\beta-1}) \cdot \omega_\beta(EMST)$, *where* H *is any low-weighted structure.*

Here $\omega(G)$ is the total length of the links in G, *i.e.*, $\omega(G) = \sum_{uv \in G} \|uv\|$, and $\omega_\beta(G)$ is the total power consumption of links in G, *i.e.*, $\omega_\beta(G) = \sum_{uv \in G} \|uv\|^\beta$. Consequently, low-weighted structure is asymptotically optimal for broadcasting among any connected structures built in a localized manner. Notice that, the above analysis is based on the assumption that every link is used during the broadcast (one-to-one communication), such as using the TDMA scheme. Even considering that the broadcast signal sent by a node can be received by all nodes in its transmission region simultaneously (one-to-all communication), the above claim is also correct. The reason is basically as follows. Let $B_s(H)$ be the total energy consumed by broadcasting on a structure H with sender s using the one-to-all communication model. Clearly, ***any*** flood-based broadcast based on a structure H consumes energy at most $\sum_{e_i \in H} e_i^\beta$ if the message received by an intermediate node v is not forwarded to its parent, *i.e.*, the node that just forwarded this message to v; and the total energy is at most $2\sum_{e_i \in H} e_i^\beta$ if an intermediate node v may also forward the message to its parent. On the other hand, the total energy $B_s(H)$ used by *any* structure H is at least $\sum_{e_i \in EMST} e_i^\beta / 12$ [45]. Thus,

$$B_s(EMST) \geq \sum_{e_i \in EMST} e_i^\beta / 12 = \omega_\beta(EMST)/12.$$

Then, if H is a low-weighted structure, we have

$$\begin{aligned} B_s(H) &\leq 2\sum_{e_i \in H} e_i^\beta = O(n^{\beta-1}) \cdot \omega_\beta(EMST) \\ &\leq 12 \cdot O(n^{\beta-1}) \cdot B_s(EMST) \end{aligned}$$

Recall that $B_s(EMST)$ is no more than a constant (≤ 8) times of the optimum in an one-to-all communication model [45, 12]. Consequently, we have the following lemma.

Lemma 2. *The broadcast based on any low-weighted structure H consumes energy at most $O(n^{\beta-1})$ times of the optimum in both one-to-one and one-to-all communication models. And the bound $O(n^{\beta-1})$ is tight.*

In summary, to enable energy efficient broadcasting, the constructed topology is also preferred to be *low-weighted*, in addition to the three properties for unicast:

4. **Low Weighted:** the total link length of final topology is within a constant factor of that of $EMST$.

Recently, several localized algorithms [30, 32] have been proposed to construct low-weighted structures, which indeed approximate the energy efficiency of EMST as the network density increasing. However, none of them is power efficient for unicast routing. In this paper we will present the first efficient distributed method to construct a planar, bounded degree spanner that is also low-weighted.

3. POWER-EFFICIENT UNICAST: SPANNER, PLANAR AND BOUNDED-DEGREE

The ultimate goal of this paper is to construct a unified topology that is power-efficient for both unicast and broadcast, in addition to be planar and have a constant bounded logical node degree. To achieve this ultimate goal, in this section, we first present a new method that can construct a power-efficient topology for unicast. We will prove that the constructed structure is a power-spanner, planar and has bounded node degree. Furthermore, it has an extra property: any two neighbors of each node are separated by at least a certain angle θ. Hereafter, we call it the Θ-*separation* property. As we will see later that this property further reduces the interference, especially when adopting directional antennas for transmission. This property also makes the proof much easier that the structure constructed in the next section is also power-efficient for broadcast.

One possible way to construct a degree-bounded planar power spanner is to apply the Yao structure on Gabriel graph, since GG is already planar and has a power stretch factor exactly 1. In [31], Li *et al.* showed that the final structure by directly applying the Yao structure on GG is a planar power spanner, called $YaoGG$, but its in-degree can be as large as $O(n)$, as in the example shown in Figure 1(b). In [42], Song *et. al* proposed two new methods to bound node degree by applying the ordered Yao structures on Gabriel graph. The structure $SYaoGG$ in [42] guarantees that there is at most one neighbor node in each of the k equal-sized cones. In this section, we will propose an improved algorithm to further reduce the medium contention by selecting less communication neighbors and separating neighbors wider.

Before we give the algorithm, we first define a concept called θ-Dominating Region.

Definition 1. θ-Dominating Region*: For each neighbor node v of a node u, the* θ-dominating region *of v is the 2θ-cone emanated from u, with the edge uv as its axis.*

Figure 2 illustrates the θ-dominating region of a node v in the transmission disk of node u. Using the concept of θ-dominating region instead of absolute cone partition in SYaoGG [42], we are able to prove that any two neighbors of each node are guaranteed to be separated by at least an angle θ. The final topology will be called $S\Theta GG$. Intuitively, the communication interference in $S\Theta GG$ will be smaller than the interference in $SYaoGG$, which is also verified later by simulations as shown in Figure 10(c) and (d).

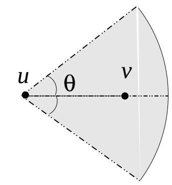


Figure 2: Node v's θ-Dominating Region with respect to node u.

Algorithm 1 $S\Theta GG$: Power-Efficient Unicast Topology

1: First, each node self-constructs the Gabriel graph GG locally. Initially, all nodes mark themselves White, *i.e.*, *unprocessed.*

2: Once a White node u has the smallest ID among all its White neighbors in $N(u)$, it uses the following strategy to select neighbors:

   1. Node u first sorts all its Black neighbors (if available) in $N(u)$ in the distance-increasing order, then sorts all its White neighbors (if available) in $N(u)$ similarly. The sorted results are then restored to $N(u)$, by first writing the sorted list of Black neighbors then appending the sorted list of White neighbors.
   2. Node u scans the sorted list $N(u)$ from left to right. In each step, it keeps the current pointed neighbor w in the list, while deletes every *conflicted* node v in the remainder of the list. Here a node v is conflicted with w means that node v is in the θ-dominating region of node w. Here $\theta = 2\pi/k$ ($k \geq 9$) is an adjustable parameter.

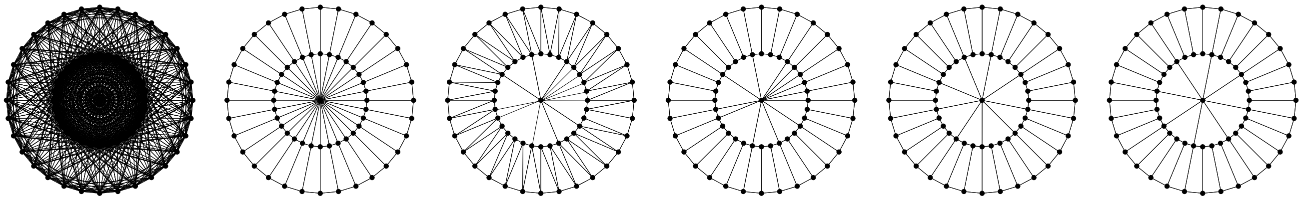
   Node u then marks itself Black, *i.e. processed*, and notifies each deleted neighboring node v in $N(u)$ by a broadcasting message UpdateN.

3: Once a node v receives the message UpdateN from a neighbor u in $N(v)$, it checks whether itself is in the nodes set for deleting: if so, it deletes the sending node u from list $N(v)$, otherwise, marks u as BLACK in $N(v)$.

4: When all nodes are processed, all selected links $\{uv | v \in N(u), \forall v \in GG\}$ form the final network topology, denoted by $S\Theta GG$. Each node can shrink its transmission range as long as it sufficiently reaches its farthest neighbor in the final topology.

The basic idea of our method is as follows. Since the Gabriel graph is planar and power-spanner, we will remove some links of GG to bound the nodal degree while not destroying the power-spanner property. The basic approach of bounding the nodal degree is to only keep some shortest link in the θ-Dominating region for every node. We process the nodes in a certain order. A node is marked White if it is unprocessed and is marked Black if it is processed. Originally all nodes are marked White. Initially, a node elects itself to start processing its neighbors if its ID[1] is smaller than all its

[1]It is not necessary to use ID here. We can also use some other mechanism to elect a certain node to perform the remaining pro-



(a) UDG (b) RNG, GG (c) BPS (d) $OrdYaoGG$ (e) $SYaoGG$ (f) $S\Theta GG$

Figure 1: Several planar power spanners for UDG shown in (a). Here $k = 9$ for constructing $SYaoGG$, $S\Theta GG$.

unprocessed logical neighbors in the Gabriel graph. Assume that a node u is to be processed. We further assume that there are already some processed logical neighboring nodes, say $v_1, \cdots, v_t$, among its neighbors in GG. It keeps the link to the closest processed neighbor, say v_1, in GG, and removes all links to all neighbors in the *θ-dominating region* of v_1. In other words, the neighbor v_1 dominates all other neighbors in its θ-dominating region. It then repeats the above procedure until no processed logical neighbors in GG are left. Assume that node u also has some unprocessed logical neighbors, *i.e.*, marked WHITE. The node u then keeps the link to the closest unprocessed neighbor, say w, in GG if there is any, and then removes the links to all neighbors in the the *θ-dominating region* of w. It then repeats the above procedure until no unprocessed neighbors in GG are left. Node u then marks itself BLACK and then informs its logical neighbors in GG about its change of status. The algorithm terminates when all nodes are marked processed. The remaining links form the final structure, called $S\Theta GG$.

In our new algorithm, a data structure will be used: $N(u)$ is the set of neighbors of each node u in the final topology, which is initialized as the set of neighbor nodes in GG. We are now ready to present our Algorithm 1, which constructs a bounded degree planar power spanner.

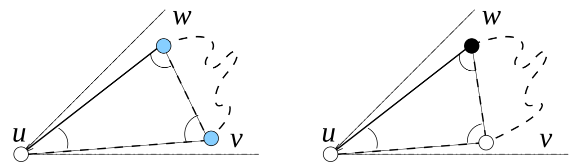
Notice that the final topology based on Yao graph, such as $SYaoGG$ [42], may vary as the choice of the direction of cones varies. Here, $S\Theta GG$ does not rely on the absolute cone partition by adopting the new concept of θ-dominating region. Hence, given the point set V, $S\Theta GG$ is unique. In addition, the average node degree, interference and transmission range of $S\Theta GG$ is expected to be smaller than $SYaoGG$ too. Furthermore, it is interesting to notice that the theoretical bound on the spanning ratio for $S\Theta GG$, that we can prove, is same as $SYaoGG$, as proved later in Theorem 4.

LEMMA 3. *Graph $S\Theta GG$ is connected if the underlying graph GG is connected. Furthermore, given any two nodes u and v, there exists a path $\{u, t_1, ..., t_r, v\}$ connecting them such that all edges have length less than $\sqrt{2}\|uv\|$.*

PROOF. We prove the connectivity by contradiction. Suppose a link uv is the shortest link in UDG whose connectivity is broken by Algorithm 1. W.l.o.g, assume the link uv is removed while processing node u, because of the existence of another node w.

As shown in Figure 3, there are only two cases (ties are broken by ID) that the link uv can be removed by node u:

cedures first. For example, we can use the RTS/CTS mechanism provided in the MAC layer to achieve this: the node that first successfully sent a RTS signal within its one-hop neighborhood will be elected. In this paper, we use ID just for the sake of an easy presentation.



(a) $\|uw\| < \|uv\|$ (b) $\|uw\| > \|uv\|$

Figure 3: Two cases when uv is removed while processing u.

Case a: $\|uw\| < \|uv\|$. Notice that $\angle vuw \le \theta < \pi/4$, hence $\|wv\| < \|uv\|$. In other words, both link wv and uw are smaller than link uv. Since there are no paths $u \leftrightsquigarrow v$ according to the assumption, either the path $u \leftrightsquigarrow w$ or $v \leftrightsquigarrow w$ is broken. That is to say, either the connectivity of wv or uw is broken. Thus, uv is not the shortest link whose connectivity is broken, it is a contradiction.

Case b: $\|uw\| > \|uv\|$. It happens only when node w is *processed* and node v is *unprocessed*. Similarly, $\angle vuw \le \theta < \pi/4 < \angle uwv$ (otherwise $\angle uvw > \pi/2$ violates the Gabriel graph property), hence $\|wv\| < \|uv\|$. Since node w is a *processed* node and node u decides to keep link uw, the link uw will be kept in $S\Theta GG$. According to assumption that u and v are not connected in $S\Theta GG$, w and v are not connected either. That is to say, uv is not the shortest link whose connectivity is broken. It is a contradiction.

This finishes the proof of connectivity. Notice that the above proof implies that the shortest link uv in UDG is kept in the final topology. Clearly, the shortest link uv is in GG. Link uv cannot be removed in our algorithm due to the case illustrated by Figure 3 (a). Assume, for the sake of contradiction, that uv is removed due to the case (b) where $\|uw\| > \|uv\|$ and w is *processed* when processing u. Then $\|wv\| < \|uv\|$ is a contradiction to that uv is the shortest link in UDG.

We then show by induction that, given any link uv in UDG, there is a path connecting them using edges with length at most $\sqrt{2}\|uv\|$. Assume uv is removed when processing u and due to the existence of link uw. We build a path connecting u and v by concatenating $u \leftrightsquigarrow w$ and $w \leftrightsquigarrow v$, as shown in Figure 3. It is not difficult to see that the longest segment of the path is less than $\sqrt{2}\|uv\|$, which occurs in case (b). In this case, the link uw must be kept because both endpoints are *processed*, and $\|uw\| < \sqrt{2}\|uv\|$. This finishes the proof. $\square$

The property that for any link uv, there is a path connecting them such that the links on the path have length at most

$\sqrt{2}\|uv\|$ is crucial for our later proof that our Algorithm 2 builds a low-weighted bounded degree planar spanner.

THEOREM 4. *The structure $S\Theta GG$ has node degree at most $k-1$ and is planar power spanner with neighbors Θ-separated. Its power stretch factor is at most $\rho = \frac{\sqrt{2}^{\beta}}{1-(2\sqrt{2}\sin\frac{\pi}{k})^{\beta}}$, where $k \geq 9$ is an adjustable parameter.*

PROOF. The proof would be similar with the proof of $SYaoGG$ in [42]. The only difference is that, we used the concept of dominating cones instead of Yao graph. While the power stretch factor remains the same theoretically, the degree bound is reduced from k to $k-1$. Obviously, the links in $S\Theta GG$ are Θ-separated, in other words, the direction of any two neighbors of a node is Θ-separated. □

Figure 1 (e) and (f) show the difference of $SYaoGG$ and $S\Theta GG$. Compared with $SYaoGG$, $S\Theta GG$ is more evenly distributed and has a lower node degree.

4. UNIFIED POWER-EFFICIENT TOPOLOGY: DEGREE-BOUNDED PLANAR SPANNER WITH LOW WEIGHT

To the best of our knowledge, so far, no communication efficient topology control algorithm has achieved all the desirable properties summarized in Section 2: *degree-bounded*, *planar*, *power spanner*, *low-weighted*. Those properties are not only interesting in terms of computational geometry, but also have important applications in wireless ad hoc networks, as shown in section 2: enable energy efficient unicast and broadcast routings in same structure. Recall that, spanner property ensures that an energy efficient path is always kept for any pair of nodes, hence it is a necessary condition to support energy efficient unicast. While low-weighted structure is optimal for broadcast among any connected structures built locally. Unfortunately, all the known spanners, including Yao [51], GG [13] and the recent developed degree-bounded planar spanners BPS [46], $SYaoGG$, $OrdYaoGG$ [42] and $S\Theta GG$, are not low-weighted. As illustrated in Figure 1, all of them will keep at least $\frac{n-1}{2}$ links between the two circles, while EMST (in Figure 5(b)) will keep only one link between them. Hence the weight of any of them is at least $O(n) \cdot w(EMST)$.

It is worth to clarify that, in this section, we are interested in finding a subgraph to enable efficient broadcast routings, *even* based on the simple-flooding method. We do *not* aim to substitute the known broadcasting protocols. In fact, the methods used in those broadcasting protocols [50, 7] can be applied on the low-weighted structures to conserve more energy. The main contribution of low-weighted structure is that it bounds the worst case performance for broadcasting.

Several known localized algorithms are given in [30, 32] to generate low-weighted graphs. In their algorithms, given a certain structure G, for any two links uv and xy of a graph G, they remove xy if xy is the longest link among quadrilateral $uvxy$. They proved that the final structures are low-weighted if G is RNG' [30] or $LMST_2$ [32]. Obviously, they are not spanners. In fact, their techniques can *not* be applied on spanner graph to bound the weight without losing its spanner property. Figure 4 illustrates an example by applying their algorithms to $S\Theta GG$. The node ID of v_i is i, $\angle v_1v_3v_4 < \theta$ and $\|v_1v_3\| > \|v_3v_4\| > \max(\|v_1v_2\|, \|v_2v_4\|)$. While constructing $S\Theta GG$, first node

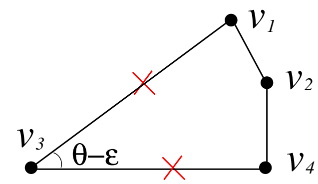
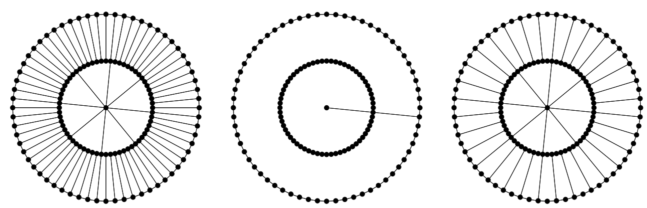


Figure 4: The graph could be disconnected if applying the previous method to build low-weighted structure on $S\Theta GG$.

v_1 selects v_1v_2 and v_1v_3 as its incident logical links and node v_2 selects v_2v_1 and v_2v_4, then node v_3 selects v_3v_1 and deletes v_3v_4. Hence $v_3v_4 \notin S\Theta GG$. If applying the rule described in [30, 32], the link v_1v_3 will also be deleted because $\|v_1v_3\| > \max(\|v_1v_2\|, \|v_2v_4\|, \|v_3v_4\|)$. Then the graph will be disconnected. Then we can conclude that simple extension of methods in [32] on top of $S\Theta GG$ does not even guarantee the connectivity, nor to say *power-spanner* property.

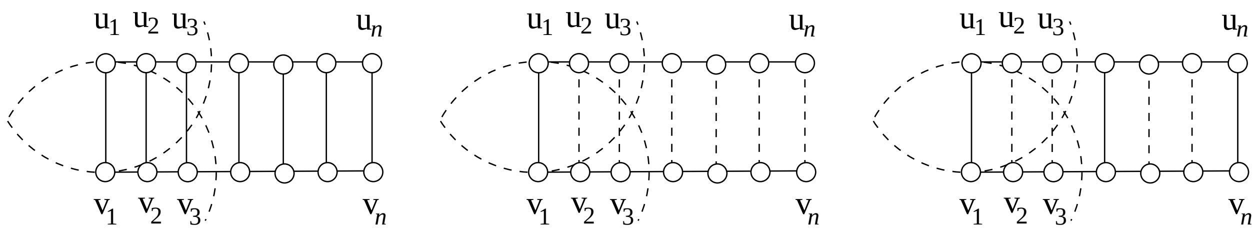
Indeed, the spanner property and low-weight property are *not* easy to be achieved at same time. Intuitively, the spanner property requires to keep more links, while the low-weight property requires to keep less links from original graph. In the following, we will describe a novel algorithm to build a low-weighted structure from $S\Theta GG$, while keeping enough links to guarantee the power efficiency. Figure 5 illustrates the difference of $LS\Theta GG$ from $S\Theta GG$ and $LMST_2$.



(a) $S\Theta GG$ (b) $LMST_2$ (c) $LS\Theta GG$

Figure 5: Three different structures.

Algorithm 2 presents our new method that constructs a bounded degree planar power-spanner that is also low-weighted. Although our algorithm produces only power-spanner here, it can be extended to produce also the length-spanner if it is needed. To get a length-spanner, we construct the structure $LDel^2$ (defined in [27]) instead of the Gabriel graph used in our algorithm. It was proved in [27] that $LDel^2$ is a planar, length-spanner, and can be constructed using only $O(n)$ messages. The basic idea of our new method is as follows. Since the graph $S\Theta GG$ is already planar, power-spanner, and has bounded-degree, we will remove some of its edges to guarantee that the resulting topology is low-weighted while not destroying the power-spanner property. Notice that removing edges will not break the planar property and the bounded-degree property. In all previous methods presented in the literature, a node x decides to remove or keep links that are incident on x, *i.e.*, it only cares about the incident edges. While, in the method presented here, a node x will decide whether to keep or remove links for not only incident links, but also the links that are incident on one of its neighbors. To guarantee a low-weight property the methods presented in [30, 32] remove



(a) original graph $S\Theta GG$ (b) graph resulted using method in [30] (c) graph based on our method

Figure 6: A sequence of links are recursively removed. Here solid and dashed links represent the links from the original graph and the dashed links represent the links that are removed by a topology control algorithm, while solid links represent the final structure constructed by a certain method. Here we assume that $\|u_iv_i\| = R$ and the ID of link u_iv_i is less than the ID of link $u_{i+1}v_{i+1}$.

some links from a certain structure such that the remaining links satisfy the *isolation property*: for each remaining link xy, the disk centered at the midpoint of xy using a radius proportional to $\|xy\|$ does not intersect with any other remaining links. They achieved this property by removing a link xy if there is another link uv such that xy is the longest link in the quadrilateral $uvyx$. However, this simple heuristic cannot guarantee the spanner property. Consider a link xy in some shortest path from s to t. See Figure 7 for an illustration. Link xy will be removed due to the existence of link uv. Link uv could also later be removed due to the existence of another link u_1v_1, which could also be removed due to the existence of another link u_2v_2, and so on. See Figure 6 (b) for an illustration of the situation where a sequence of links will be removed: all links u_iv_i, for $i \geq 2$ will be removed. Consequently, the shortest path connecting nodes u_n and v_n could be arbitrarily long in the resulting graph.

Thus, instead of blindly removing all such links xy whenever it is the longest link in a quadrilateral $uvyx$, we will keep such a link when some links in its certain neighborhood have been removed. To do so, among all links from a graph, such as $S\Theta GG$, that is planar, bounded-degree, power-spanner, we *implicitly* define an independent set of links. A link is in this independent set, which will be kept at last, if it has the smallest ID among unselected links from its neighborhood. Specifically, we implicitly define a virtual graph G' over all links in $S\Theta GG$: the vertex set of G' is the set of all links in $S\Theta GG$ and two links xy and uv of $S\Theta GG$ are connected in G' if one end-point of uv is in the transmission range of one end-point of link xy (they will interfere with each other if transmit simultaneously). For example, the links u_1v_1 and u_3v_3 are not independent in network topology illustrated by Figure 6 (a); while the links u_1v_1 and u_nv_n are independent. Notice that links u_1v_1 and u_1u_2 are independent since they do not form a four vertices convex hull. Notice that in our method presented later, we did not explicitly define such graph G', nor compute the maximal independent set of such graph G' explicitly. We will prove that the selected independent set of links in $S\Theta GG$ indeed is low-weighted and still preserves the power-spanner property, although with a larger power spanning ratio. Our method will keep link u_1v_1 since it has the smallest ID among all links that are not independent. When link u_1v_1 is kept, all links that are not independent (here are u_2v_2 and u_3v_3) will be removed. Then link u_4v_4 will be kept. The above procedure will be repeated until all links are processed. The final structure resulted from our method is illustrated by Figure 6 (c).

Obviously, the construction is consistent for two endpoints of each edge: if an edge uv is kept by node u, then it is also kept by node v. The ID of a link uv is defined as following: $ID(uv) = \{\|uv\|, \min(ID(u), ID(v)), \max(ID(u), ID(v))\}$. As we will see later that the number 3 in criterion of Algorithm 2

$$\|xy\| > \max(\|uv\|, 3\|ux\|, 3\|vy\|)$$

is carefully selected.

Algorithm 2 Construct $LS\Theta GG$: Planar Spanner with Bounded Degree and Low Weight

1: All nodes together construct the graph $S\Theta GG$ in a localized manner, as described in Algorithm 1. Then, each node marks its incident edges in $S\Theta GG$ *unprocessed*.

2: Each node u locally broadcasts its incident edges in $S\Theta GG$ to its one-hop neighbors and listens to its neighbors. Then, each node x can learn the existence of the set of 2-hop links $E_2(x)$, which is defined as follows: $E_2(x) = \{uv \in S\Theta GG \mid u \text{ or } v \in N_{UDG}(x)\}$. In other words, $E_2(x)$ represents the set of edges in $S\Theta GG$ with at least one endpoint in the transmission range of node x.

3: Once a node x learns that its *unprocessed* incident edge xy has the smallest ID among all *unprocessed* links in $E_2(x)$, it will delete edge xy if there exists an edge $uv \in E_2(x)$ (here both u and v are different from x and y), such that $\|xy\| > \max(\|uv\|, 3\|ux\|, 3\|vy\|)$; otherwise it simply marks edge xy *processed*. Here assume that $uvyx$ is the convex hull of u, v, x and y. Then the link status is broadcasted to all neighbors through a message UPDATESTATUS(XY).

4: Once a node u receives a message UPDATESTATUS(XY), it records the status of link xy at $E_2(u)$.

5: Each node repeats the above two steps until all edges have been *processed*. Let $LS\Theta GG$ be the final structure formed by all remaining edges in $S\Theta GG$.

THEOREM 5. *The structure $LS\Theta GG$ is a degree-bounded planar spanner. It has a power spanning ratio $2\rho + 1$, where ρ is the power spanning ratio of $S\Theta GG$. The node degree is bounded by $k - 1$ where $k \geq 9$ is a customizable parameter in $S\Theta GG$.*

PROOF. The degree-bounded and planar properties are obviously derived from the $S\Theta GG$ graph, since we do not add any links in Algorithm 2. To prove the spanner property, we only need to show that the two endpoints of any deleted link $xy \in S\Theta GG$ are still connected in $LS\Theta GG$ with a constant spanning ratio path. We will prove it by induction on the length of deleted links from $S\Theta GG$.

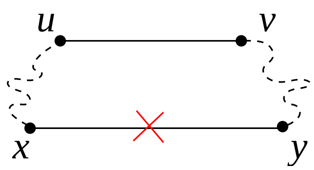


Figure 7: The path between x and y is at most $(2\rho + 1)\|xy\|$ in $LS\Theta GG$ if $xy \in S\Theta GG$.

Assume xy is the shortest link of $S\Theta GG$ which is deleted by Algorithm 2 because of the existence of link uv with smaller length. Obviously, path $x \rightsquigarrow y$ can be constructed through the concatenation of path $x \rightsquigarrow u$, link uv and path $v \rightsquigarrow y$, as shown in Figure 7. Since $\|xy\| > \max(\|ux\|, \|vy\|)$ and link xy is the shortest among deleted links in Algorithm 2, we have $p(x \rightsquigarrow u) < \rho\|ux\|^{\beta}$ and $p(v \rightsquigarrow y) < \rho\|vy\|^{\beta}$. Hence, $p(x \rightsquigarrow y) < \|uv\|^{\beta} + \rho\|ux\|^{\beta} + \rho\|vy\|^{\beta} < (2\rho + 1)\|xy\|^{\beta}$.

Suppose all the i-th ($i \leq t-1$) deleted shortest links of $S\Theta GG$ have a path connecting their endpoints with spanning ratio $2\rho + 1$. For the t-th deleted shortest link $xy \in S\Theta GG$, according to Algorithm 2, it must have been deleted because of the existence of a link uv: such that $\|xy\| > \max(\|uv\|, 3\|ux\|, 3\|vy\|)$ in a convex hull $uvyx$. Now, we have $p(x \rightsquigarrow u) < (2\rho+1)\|ux\|^{\beta}$ and $p(v \rightsquigarrow y) < (2\rho + 1)\|vy\|^{\beta}$. Thus,

$$\begin{aligned} p(x \rightsquigarrow y) &= \|uv\|^{\beta} + p(u \rightsquigarrow x) + p(v \rightsquigarrow y) \\ &< \|uv\|^{\beta} + (2\rho+1)\|ux\|^{\beta} + (2\rho+1)\|vy\|^{\beta} \\ &< \|xy\|^{\beta} + (2\rho+1)(\|xy\|/3)^{\beta} + (2\rho+1)(\|xy\|/3)^{\beta} \\ &\leq (2\rho+1)\|xy\|^{\beta} \end{aligned}$$

Thus, $LS\Theta GG$ has a power spanning ratio $\leq 2\rho + 1$. $\square$

We then show that graph $LS\Theta GG$ is low-weighted. To study the total weight of this structure, inspired by the method proposed in [30], we will show that the edges in $LS\Theta GG$ satisfy the *isolation property* [10].

Theorem 6. *The structure $LS\Theta GG$ is low-weighted.*

See the appendix for the proof. We continue to analyze the communication cost of Algorithm 1 and 2. First, clearly, building GG in Algorithm 1 can be done using only n messages: each message contains the ID and geometry position of a node. Second, to build $S\Theta GG$, initially, the number of edges, say p, in Gabriel Graph is $p \in [n, 3n-6]$ since it is a planar graph. Remember that we will remove some edges from GG to bound the logical node degree. Clearly, there are at most $2n$ such removed edges since we keep at least $n-1$ edges from the connectivity of the final structure. Thus the total number of messages, say q, used to inform the deleted edges from GG is at most $q \in [0, 2n]$. Notice that $p - q$ is the edges left in the final structure, which is at least $n-1$ and at most $3n-6$. Thirdly, in the marking process described in Algorithm 2, the communication cost of broadcasting its incident edges (or its neighbors) and updating link status are both $2(p-q)$. Therefore the total communication cost is $n + 4p - 3q \in [5n, 13n]$. Then the following theorem directly follows.

Theorem 7. *Assuming that both the ID and the geometry position can be represented by $\log n$ bits each, the total number of messages during constructing the structure $LS\Theta GG$ is in the range of $[5n, 13n]$, where each message has at most $O(\log n)$ bits.*

Compared with previous known low-weighted structures [30, 32], $LS\Theta GG$ not only achieves more desirable properties, but also costs much less messages during construction. To construct $LS\Theta GG$, we only need to collect the information $E_2(x)$ which costs at most $6n$ messages. Our algorithm can be generally applied to any known degree-bounded planar spanner to make it low-weighted while keeping all its previous properties, except increasing the spanning ratio from ρ to $2\rho + 1$ theoretically.

5. EXPECTED INTERFERENCE IN RANDOM NETWORKS

This section is devoted to study the average physical node degree of our structure when the wireless nodes are distributed according to a certain distribution. For average performance analysis, we consider a set of wireless nodes distributed in a two-dimensional unit square region. The nodes are distributed according to either the uniform random point process or homogeneous Poisson process. A point set process is said to be a *uniform random point process*, denoted by $\mathcal{X}_n$, in a region Ω if it consists of n independent points each of which is uniformly and randomly distributed over Ω. The standard probabilistic model of *homogeneous Poisson process* is characterized by the property that the number of nodes in a region is a random variable depending only on the area (or volume in higher dimensions) of the region. In other words,

- The probability that there are exactly k nodes appearing in any region Ψ of area A is $\frac{(\lambda A)^k}{k!} \cdot e^{-\lambda A}$.
- For any region Ψ, the conditional distribution of nodes in Ψ given that exactly k nodes in the region is *joint uniform.*

Definition 2. *Given a structure H, the adjusted transmission range $r_H(u)$ is defined as $\max_{uv \in H} \|uv\|$, i.e., the longest edge of H incident on u. The physical node degree of u in H is defined as the number of nodes inside the disk $disk(u, r_H(u))$. The node interference, denoted by $I_H(u)$, of a node u in a structure H is simply the physical node degree of u. The maximum node interference of a structure H is defined as $\max_u I_H(u)$. The average node interference of a structure H is defined as $\sum_u I_H(u)/n$.*

Theorem 8. *For a set of nodes produced by a Poisson point process with density n, the expected maximum node interference of* any *connected structure, e.g., EMST, GG, RNG, Yao and $LS\Theta GG$, is at least $\Theta(\log n)$.*

Proof. Let $d_n(H)$ be the longest edge of a structure H of n points placed independently in 2-dimensions according to standard poisson distribution with density n. Obviously, $d_n(EMST)$ is the smallest among *all* connected structures H. For simplicity, let $d_n = d_n(EMST)$. In [37], they showed that

$$\lim_{n \to \infty} P_r(n\pi d_n^2 - \log n \leq \alpha) = e^{-e^{-\alpha}}.$$

Notice that the probability $P_r(n\pi d_n^2 - \log n \leq \log n)$ will be sufficiently close to 1 when n goes to infinity, while the probability $P_r(n\pi d_n^2 - \log n \leq -\log\log n)$ will be sufficiently close to 0 when n goes to infinity. That is to say, with high probability, $n\pi d_n^2$ is in the range of $[\log n - \log\log n, 2\log n]$.

Given a region with area A, let $m(A)$ denote the number of nodes inside this region by a Poisson point process with density δ. According to the definition of Poisson distribution, $P_r(m(A) = k) = \frac{e^{-\delta A}(\delta A)^k}{k!}$. Thus, the expected number of nodes lying inside a region with area A is $E(m(A)) = \delta A$. For a Poisson process with density n, let uv be the longest edge of the Euclidean minimum spanning tree, and $d_n = \|uv\|$. Then, the expectation of the number of nodes that fall inside $disk(u, d_n)$ is $E(m(\pi d_n^2)) = n\pi d_n^2$, which is larger than $\log n$ almost surely when n goes to infinity. That is to say, the expected maximum interference of Euclidean MST is $\Theta(\log n)$ for a set of nodes produced according to a Poisson point process.

Since $d_n \leq d_n(H)$ for any connected structure H, the expected maximum node interference of any connected structure H, *e.g.*, GG, RNG, Yao, and $LS\Theta GG$, is at least $\Omega(\log n)$. Thus, all commonly used structures for topology control in wireless ad hoc networks have a large maximum node interference even for *randomly* deployed nodes. □

It is not difficult to show that the above theorem is also true when the nodes are distributed according to a uniform random distribution. Our following analysis will show that the average interference of all nodes of these structures is small for a randomly deployed network.

THEOREM 9. *For a set of nodes produced by a Poisson point process with density n, the expected average node interferences of EMST and RNG are bounded from above by some constants.*

PROOF. Consider a set V of wireless nodes produced by Poisson point process. Given a structure G, the interference $I_G(u_i)$ is the number of nodes inside the transmission region of node u_i. Here the transmission region of node u_i is a disk centered at u_i with radius $r_i = \max_{u_i v \in G} \|u_i v\|$. Hence, the expected average node interference is

$$E(\frac{\sum_{i=1}^n I_G(u_i)}{n}) = \frac{1}{n}E(\sum_{i=1}^n I_G(u_i)) = \frac{1}{n}\sum_{i=1}^n E(I_G(u_i))$$
$$= \frac{1}{n}\sum_{i=1}^n E(m(\pi r_i^2)) = \frac{1}{n}\sum_{i=1}^n (n\pi r_i^2) \leq 2\sum_{e_i \in G}(\pi e_i^2).$$

The last inequality follows from the fact that r_i is the length of some edge in G and each edge in G can be used by at most two nodes to define its radius r_i.

Let e_i, $1 \leq i \leq n-1$ be the length of all edges of the EMST of n points inside a unit disk. It was shown in [45] that $\sum_{e_i \in EMST} e_i^2 \leq 12$. Thus, the expected average node interference of the structure EMST is

$$E(\frac{\sum_{i=1}^n I_{EMST}(u_i)}{n}) \leq 2\sum_{e_i \in EMST}(\pi e_i^2) \leq 24\pi.$$

For RNG graph, we define a diamond for each segment. The *open diamond* subtended by a line segment uv, denoted by $D(uv, \gamma)$, is the rhombus with sides of length $\|uv\|/(2\cos\gamma)$, where $0 \leq \gamma \leq \pi/3$ is a parameter. Similar to the proof of [45], we can show that the diamonds $D(uv, \pi/6)$ do not overlap and $\sum_{e_i \in RNG} e_i^2 \leq 8\pi/\sqrt{3}$. This implies that

$$E(\frac{\sum_{i=1}^n I_{RNG}(u_i)}{n}) \leq 2\sum_{e_i \in RNG}(\pi e_i^2) \leq 16\pi^2/\sqrt{3}.$$

This finishes our proof. □

THEOREM 10. *The expected average node interference of $LS\Theta GG$ is bounded from above by a constant.*

PROOF. We prove it by showing that in $LS\Theta GG$, all the diamonds $D(uv, \gamma)$ subtended from each link segment $uv \in LS\Theta GG$ do not overlap with each other, where $\sin 2\gamma = \frac{1}{3}$. Figure 8 illustrates the basic idea of the proof. For any

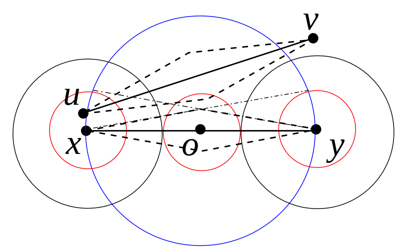


Figure 8: All diamonds do not overlap.

two segments uv and xy, we can show that either the angle between them is at least 2γ (implies that two diamonds $D(uv, \gamma)$ and $D(xy, \gamma)$ do not overlap), or the distance between them is far enough to separate these two diamonds. The detail of the proof is omitted here, since it is not difficult to verify, although it is tedious.

It is easy show that the total area of these diamonds is $\frac{\tan\gamma}{2}\sum_{e_i \in LS\Theta GG} e_i^2 \simeq 0.084\sum_{e_i \in LS\Theta GG} e_i^2$. Then we can show that $\sum_{e_i \in LS\Theta GG} e_i^2 \leq 12\pi$. Thus, the expected average node interference is at most $2\sum_{e_i \in LS\Theta GG} e_i^2 \leq 24\pi$. □

6. DYNAMIC UPDATE

The methods presented so far have assumed that the wireless nodes are static. This is true for some certain wireless networks such as the wireless sensor networks. Obviously, dynamic maintenance of the topology is an important issue for a dynamic wireless ad hoc network, after the construction of the unified energy-efficient topology. Two major events may cause the topology "obsoleted": 1) *topology changes* due to node moving, node joining or leaving, node failure; and 2) *distance changes* when nodes are moving. Notice that the distance clearly change frequently. It will be very expensive if we respond quickly to all possible distance changes. Therefore, a dynamic update method for the network topology will respond to the topology change spontaneously: topology should be updated carefully when a certain used link in the topology is no longer a physical link in the network. On the other hand, it will respond to the distance change only when the change is over some threshold. Assume that there is a timer set for topology maintenance: when the timer expires, each node will check whether it needs to update the topology. We propose to set two control parameters $\alpha_2 > 1 > \alpha_1 > 0$. Assume that the nodes are moving and let $\tilde{u}$ be the new position of a node u after the timer for topology-maintenance expires. Let $\overline{u}$ be the position of node u when the last topology-maintenance is performed and $\overline{LS\Theta GG}$ be the corresponding updated topology. In our topology control implementation, a node u will not perform the topology update if for every *logical* neighbor v, we have

$$\alpha_1 \cdot \|\overline{uv}\| \leq \|\widetilde{u}\widetilde{v}\| \leq \alpha_2 \cdot \|\overline{uv}\|.$$

Notice that if this condition is hold for all nodes, then the "old" topology $\overline{LS\Theta GG}$ we currently have is still a low-weighted, bounded degree, planar spanner for the current node configuration. The planar property and the bounded degree property clearly hold. We only need to show that that structure $\overline{LS\Theta GG}$ is still low-weighted and a power-spanner for the new node positions. We first prove that it is still a power-spanner, although not the same structure if we apply our method using the current positions of all nodes. For the presentation convenience, let $\widetilde{UDG}$ be the network formed by all nodes u at their current position $\widetilde{u}$; Let $\overline{UDG}$ be the network formed by all nodes u at their previous position $\overline{u}$ when the last topology updating is performed.

THEOREM 11. *Structure $\overline{LS\Theta GG}$ is a power-spanner for the new network modelled by $\widetilde{UDG}$.*

PROOF. Consider any link uv. In the current network $\widetilde{UDG}$, there is a shortest path connecting them with the minimum power consumption. For each link xy on this shortest path, we have $\|\overline{xy}\| \leq \|\widetilde{x}\widetilde{y}\|/\alpha_1$. Notice that it is possible that link xy is not in the topology $\overline{LS\Theta GG}$. Since $\overline{LS\Theta GG}$ is a power-spanner for $\overline{UDG}$, there must exist a path Π in $\overline{LS\Theta GG}$ connecting x and y using total power at most $(2\rho+1) \cdot \|\overline{xy}\|^\beta$, which is at most $\frac{2\rho+1}{\alpha_1^\beta} \cdot \|\widetilde{x}\widetilde{y}\|^\beta$. Notice that the length of each edge of Π may be different now in current network $\widetilde{UDG}$ and the old network $\overline{UDG}$. Obviously, $\|\widetilde{a}\widetilde{b}\|/\alpha_2 \leq \|\overline{ab}\|$ for each edge ab of the logical path Π. Consequently, the total power for the topological path Π in network $\widetilde{UDG}$ will be at most

$$\alpha_2^\beta \cdot \frac{2\rho+1}{\alpha_1^\beta} \cdot \|\widetilde{x}\widetilde{y}\|^\beta.$$

Thus, the topological structure $\overline{LS\Theta GG}$ still has a power-spanning ratio at most $\alpha_2^\beta \cdot \frac{2\rho+1}{\alpha_1^\beta}$ for the current network modelled by $\widetilde{UDG}$. This finishes the proof. $\square$

We then show that the structure $\overline{LS\Theta GG}$ is still low-weighted, *i.e.*, the total edge length of the topological structure $\overline{LS\Theta GG}$ under the new positions of nodes is still within a constant factor of the total edge length of the Euclidean minimum spanning tree of the new network modelled by $\widetilde{UDG}$.

THEOREM 12. *Structure $\overline{LS\Theta GG}$ is still low-weighted for the new network modelled by $\widetilde{UDG}$.*

PROOF. For simplicity, we use $\overline{EMST}$ to denote the Euclidean minimum spanning $EMST(\overline{UDG})$ of the network $\overline{UDG}$. We define $\widetilde{EMST}$ similarly. Since $\overline{LS\Theta GG}$ is originally low-weighted for network modelled by $\overline{UDG}$, there is a constant c such that

$$\sum_{xy \in \overline{LS\Theta GG}} \|\overline{xy}\| = \omega(\overline{LS\Theta GG}) \leq c \cdot \sum_{uv \in \overline{EMST}} \|\overline{uv}\|.$$

Clearly, $\sum_{xy \in \overline{LS\Theta GG}} \|\widetilde{x}\widetilde{y}\| \leq \alpha_2 \cdot \sum_{xy \in \overline{LS\Theta GG}} \|\overline{xy}\|$. Let's consider the tree $\widetilde{EMST}$. Obviously, we have

$$\begin{aligned} &\textstyle\sum_{st \in \widetilde{EMST}} \|\widetilde{s}\widetilde{t}\| \geq \sum_{st \in \widetilde{EMST}} \alpha_1 \cdot \|\overline{st}\| \\ &= \alpha_1 \cdot \textstyle\sum_{st \in \widetilde{EMST}} \|\overline{st}\| \geq \alpha_1 \cdot \sum_{uv \in \overline{EMST}} \|\overline{uv}\| \end{aligned}$$

The last inequality is due to tree $\overline{EMST}$ is the minimum spanning tree when each node u has a position $\overline{u}$. Thus, the total edge length of $\overline{LS\Theta GG}$ with new nodes' positions is

$$\begin{aligned} & \sum_{xy \in \overline{LS\Theta GG}} \|\widetilde{x}\widetilde{y}\| \leq \alpha_2 \cdot \sum_{xy \in \overline{LS\Theta GG}} \|\overline{xy}\| \\ \leq & \quad \alpha_2 \cdot c \cdot \sum_{uv \in \overline{EMST}} \|\overline{uv}\| \leq \alpha_2 \cdot c \cdot \frac{1}{\alpha_1} \sum_{st \in \widetilde{EMST}} \|\widetilde{s}\widetilde{t}\| \\ = & \quad \frac{\alpha_2}{\alpha_1} \cdot c \cdot \omega(\widetilde{EMST}) \end{aligned}$$

This finishes our proof. $\square$

Usually, there are two kinds of update methods: on-demand update or periodical update. Most of the existing dynamic updating algorithms are invoked periodically, while some algorithms perform the updating only when it is required (i.e., on-demand). Our algorithm can adapt and combine both of these two update methods. If no major topology changes or no remarkable distance changes, no update will be performed until some pre-set timer expires. In other words, we perform our algorithm periodically with a pre-set time. The time could be set quite long depending on the mobility pattern of the wireless network. This kind of global update also insures the load balance throughout the network. But for some major topology change (such as a broken physical link used in the topology) or tremendous change of distance, an on-demand update will be performed.

7. PERFORMANCE ON RANDOM NETWORKS

In this section we evaluate the performance of our new energy efficient unicast and broadcast topology $S\Theta GG$ by conducting simulations. In our simulations, we randomly generate a set V of n wireless nodes and $UDG(V)$, then test the connectivity of $UDG(V)$. If it is connected, we construct different topologies on $UDG(V)$, including our new topology $S\Theta GG$ and some other well-known planar topologies including GG [13], RNG [43], and $SYaoGG$ [42]. Then we measure the sparseness, the power efficiency and the interference of these topologies.

In the simulation results presented here, we generate n random wireless nodes in a 16×16 unit squares; the parameter k is set to 9 when we construct $SYaoGG$ and $S\Theta GG$; the transmission range of each node is set to 4 unit. Typically, a unit represents about 50 meters here. We test the power efficiency, and node degree of these planar structures by varying the node number n from 30 to 360. For each number $n = 30i$, $1 \leq i \leq 12$, 500 vertex sets are generated. Given a sampled network, we will first compute the quality measure for this graph, *e.g.*, we compute the average (and the maximum) spanning ratio of the spanning ratios of all pairs of nodes; we compute the average (and the maximum) (physical and logical) node degree of all nodes. We then compute the average of these performances (*e.g.*, average spanning ratio, average node degree, maximum node degree) over all these 500 randomly sampled networks.

7.1 Power Efficiency for Unicast

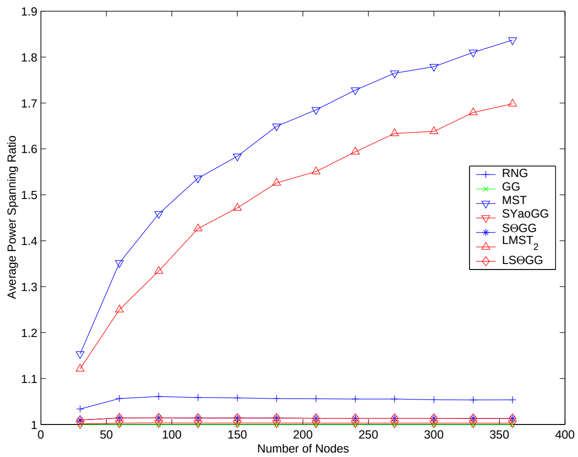


Figure 9: Average power spanning ratio of different topologies.

The most important design metric of wireless network topology is perhaps the power efficiency, as it directly affects both the node and the network lifetime. We first study power stretch factors of all structures, which are summarized in Figure 9. In our simulations we set power attenuation constant $\beta = 2$. It shows that all power spanners (GG, $SYaoGG$, $S\Theta GG$, $LS\Theta GG$) indeed have a small power spanning ratio in practice: less than 1.021, while RNG, $LMST_2$, $EMST$ are less power efficient as proved. Notice that RNG has a length spanning ratio $\Omega(\sqrt{\frac{\log n}{\log\log n}})$ even for n randomly distributed nodes [4], which implies that the length spanning ratios of $LMST_2$, $EMST$ are also at least $\Omega(\sqrt{\frac{\log n}{\log\log n}})$ since RNG contains them as subgraphs. Similar proofs can show that these structures also have arbitrarily large power spanning ratios (in the order of $\Omega(\sqrt{\frac{\log n}{\log\log n}})$) for n nodes randomly distributed in a square region. The proof details are omitted here due to space limit. The curves in Figure 9 do show such a trend of increasing power spanning ratios for structures EMST and LMST$_2$. Hence, for unicast application, we only need to compare the performances among power spanners. The average power stretch factors of $LS\Theta GG$ are at the same level of those of GG though $LS\Theta GG$ is sparser and low-weighted.

7.2 Number of Communication Neighbors

In unicast routings, each node is preferred to have a bounded number of communication neighbors. Otherwise a node with a large logical degree may have to communicate with many nodes directly. This increases the interference and the overhead at this node. The overhead could be caused by maintaining the status of these logical neighbors locally. The average and maximum logical node degrees of each topology are shown in Figure 10 (a) and (b). It shows that $S\Theta GG$ and $LS\Theta GG$ have less number of edges (average logical degrees) than $SYaoGG$ and GG. Our new structure is sparser than previous structures with bounded power-spanning ratios, but denser than other structures (RNG, EMST, LMST$_2$). The increasing/decreasing values as n increases in Figures 10 (b), (d) and (f) are caused by the small number of simulations. Notice that, to get the same confidence level, measuring the maximum value needs more simulations than measuring the average value.

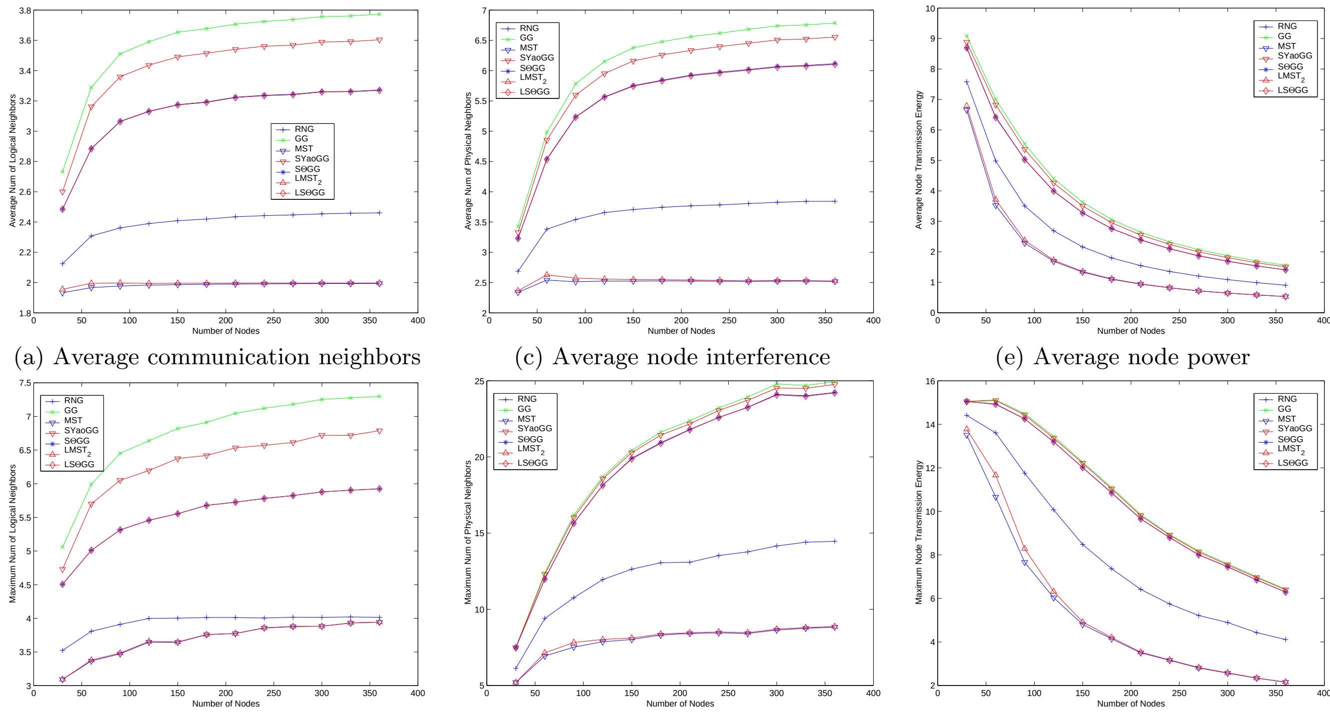
7.3 Interference

Beside the logical node degrees, we are also interested in the *physical node degree* (or called node interference) that is defined as follows. For each node u, let $uv = L_H(u)$ be the longest link incident in u in a structure H. The node interference of u is defined as the number of nodes w with $\|uw\| \leq \|uv\|$. This is the total number of nodes that could cause direct interference with u. The average and maximum node interference of each topology are shown in Figure 10 (c) and (d). They are higher than the logical node degrees as expected, however they follow the same pattern of curves. The average node interference increases first when the number of nodes increases, then it becomes stable. The average node interferences of $LS\Theta GG$, $S\Theta GG$, $SYaoGG$ are indeed bounded, which are around 6, 6, 6.5 in our simulations; the average node interferences of RNG, EMST, and LMST$_2$ are smaller but they are not efficient structures. The maximum node interference increases slightly while the number of wireless nodes grows: it follows the curve of $O(\ln n)$ as we proved. As predicted in section 3, both average and maximum node interference of $S\Theta GG$ are lower than $SYaoGG$.

7.4 Performance for Broadcasting

After forming a sparse structure, say H, each node can shrink its transmission energy as long as it is enough to cover the longest adjacent neighbor in the structure. By this way, we define the node transmission power for each node u in a structure as the minimum power needed to support its longest link $L_H(u) = uv$ in H. Here we assume that the node transmission energy of u is set as $\|uv\|^{\beta}$. Recall the discussion in section 2.2, once a structure H is constructed, the broadcast is a simple flooding on top of H: every node will forward the received broadcast message (from one of its logical neighbors) once to all its other logical neighbors in the structure. Hence minimum-energy broadcast is to decide the logical neighbors (thus the transmission range) of each node, so that the total energy consumed during broadcasting is minimized.

For broadcast we will not study the total energy consumption of all nodes since this depends on the number of nodes: more nodes will consume more energy, which makes it difficult to study the overall performance of a structure when the number of nodes varies. Instead, we will concentrate on the power consumed by individual nodes. The average and the maximum node transmission energy of each topology are shown in Figure 10 (e) and (f), which decreases as the network density increases as expected. The power assignment based on $LS\Theta GG$ is only slightly larger than RNG, which has been widely used for broadcasting previously; the reason is that $LS\Theta GG$ need more links to guarantee small power spanning ratio theoretically. The structure $LS\Theta GG$ produces the smallest average node power for broadcast among all structures with theoretical guaranteed performance for unicast and broadcast. Our theoretical results are corroborated in the simulations: low-weighted structure is indeed close to optimal for broadcast among all locally constructed structures. Moreover, simulation results in all charts also show that the performances of our new topologies $LS\Theta GG$ are stable when the number of nodes changes.



(a) Average communication neighbors (c) Average node interference (e) Average node power

(b) Maximum communication neighbors (d) Maximum node interference (f) Maximum node power

Figure 10: The average and maximum performances of various structures.

8. CONCLUSION

Energy conservation is critical to the network performance in wireless ad hoc networks. Topology control has drawn significant research interests from different approaches for energy conservation in wireless ad hoc networks. In this paper, we proposed an efficient algorithm in which all wireless nodes maintain a network topology, called $LS\Theta GG$, which is the first known single structure to support both energy efficient unicast and broadcast. We gave distributed method to construct it with $5n$ to $13n$ messages. We proved that, for unicast, it has following attractive properties: power spanner, bounded node degree, planar, and low average interference. Furthermore, the total energy of broadcast based on this structure is also within a constant factor of the power consumption of broadcast based on any *locally* constructed topology, although it is not within a constant factor of the *global* optimum. Previous known communication efficient topology control algorithms can only achieve part of those nice properties, especially, none of them can support both efficient unicast and broadcast simultaneously.

There are still lots of challenging questions we did not address in this paper. First of all, throughout this paper, we assumed that the emission power is the major component of the power consumption. In some devices, the emission power is at the same level of the power needed for being idle or to receive packets. It is then necessary to design a structure with theoretically proven worst case performance under this new energy model when the receiving power is not negligible. Secondly, an implicit assumption of the our power model is that each node can adjust its power to any specific value. In practice, the transmitter has to choose among a set of given discrete power values. It is already known that under this discrete power model, the minimum energy broadcast problem is still NP-hard [33] and a method with approximation ratio $O(n^\epsilon)$ was proposed in [33] (the ratio is $O(\log^3 n)$ if identical nodes are used). Then a problem remaining is to close the gap between this huge ratio and the constant approximation ratio [45] when the power is continuously adjustable. Thirdly, although the algorithms proposed in this paper use $O(n)$ messages, each with $O(\log n)$ bits, they are not *localized* algorithms. Recall that, a distributed algorithm is localized if it runs in constant time independent of the size of the network [34, 23]. It would be interesting to study what kind of properties can be achieved locally (as in [34]), and what kind of properties cannot be achieved locally (as in [23]). If a certain combined property cannot be achieved locally, what will be the best achievable trade-off between time and approximation ratio. Currently, the locally achievable geometry properties include: planar spanner [27], bounded degree spanner [31], bounded degree and low weight [32], k-fault tolerant [28, 26].

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10. REFERENCES

[1] I.F Akyildiz, W. Su, Y. Sankarasubramaniam, and E.Cayirci. A survey on sensor networks. *IEEE Communications Magazine*, 40(8):102–114, August 2002.

[2] P. Bose, P. Morin, I. Stojmenovic, and J. Urrutia. Routing with guaranteed delivery in ad hoc wireless networks.

ACM/Kluwer Wireless Networks, 7(6):609–616, 2001. 3rd Int. DialM Workshop, 1999, 48-55.

[3] Prosenjit Bose, Joachim Gudmundsson, and Michiel Smid. Constructing plane spanners of bounded degree and low weight. In *Proceedings of European Symposium of Algorithms*, 2002.

[4] P. Bose, L. Devroye, W. Evans and D. Kirkpatrick, On the spanning ratio of Gabriel graphs and beta-skeletons, In *Proceedings of the Latin American Theoretical Infocomatics*, 2002,

[5] Martin Burkhart, Pascal Von Rickenbach, Roger Wattenhofer, and Aaron Zollinger. Does topology control reduce interference. In *ACM MobiHoc*, 2004.

[6] Gruia Calinescu. Computing 2-hop neighborhoods in ad hoc wireless networks, 2003. AD-HOC NetwOrks and Wireless Conference.

[7] J. Cartigny, D. Simplot, and I. Stojmenovic. Localized minimum-energy broadcasting in ad-hoc networks. In *IEEE INFOCOM*, 2003.

[8] Cheng, X., Thaeler, A., Xue, G., and Chen, D., TPS: A time-based positioning scheme for outdoor wireless sensor networks. In *IEEE INFOCOM*. 2004.

[9] A. Clementi, P. Crescenzi, P. Penna, G. Rossi, and P. Vocca. On the complexity of computing minimum energy consumption broadcast subgraphs. In *18th Annual Symposium on Theoretical Aspects of Computer Science, LNCS 2010*, pages 121–131, 2001.

[10] Gautam Das, Giri Narasimhan, and Jeffrey Salowe. A new way to weigh malnourished euclidean graphs. In *ACM SODA*, pages 215–222, 1995.

[11] D.P. Dobkin, S.J. Friedman, and K.J. Supowit. Delaunay graphs are almost as good as complete graphs. *Discrete Computational Geometry*, pages 399–407, 1990.

[12] M. Flammini, R. Klasing, A. Navarra, S.Perennes, Improved approximation results for the Minimum Energy Broadcasting Problem, *ACM DIALM-POMC*, 2004

[13] K.R. Gabriel and R.R. Sokal. A new statistical approach to geographic variation analysis. *Systematic Zoology*, 18:259–278, 1969.

[14] W. R. Heinzelman, J. Kulik, and H. Balakrishnan. Adaptive protocols for information dissemination in wireless sensor networks. In *ACM MobiCom*, pages 174–185, Seattle, Washington, August 1999.

[15] Hu, L and Evans, D. Localization for mobile sensor networks. In *ACM MobiCom*. 2004.

[16] C. Intanagonwiwat, R. Govindan, D. Estrin, J. Heidemann, and F. Silva. Directed diffusion for wireless sensor networking. *IEEE/ACM Transactions on Networking*, 11(1), Feburary 2003.

[17] D. B. Johnson, D. A. Maltz, Y.-C. Hu, and J. G. Jetcheva. The dynamic source routing protocol for mobile ad hoc networks. IETF Internet Draft, November 2001. draft-ietf-manet-dsr-06.txt.

[18] Brad Karp and H.T. Kung. GPSR: Greedy perimeter stateless routing for wireless networks. In *ACM MobiCom*, 2000.

[19] J. M. Keil and C. A. Gutwin. Classes of graphs which approximate the complete euclidean graph. *Discrete Computational Geometry*, 7:13–28, 1992.

[20] J.M. Keil and C.A. Gutwin. The Delaunay triangulation closely approximates the complete euclidean graph. In *Proc. 1st Workshop Algorithms Data Structure (LNCS 382)*, 1989.

[21] L. M. Kirousis, E. Kranakis, D. Krizanc, and A. Pelc. Power consumption in packet radio networks. *Theoretical Computer Science*, 243:289–305, 2000.

[22] L. Kleinrock and J. Silvester. Optimum transmission radii for packet radio networks or why six is a magic number. In *Proceedings of the IEEE National Telecommunications Conference*, pages 431–435, 1978.

[23] Fabian Kuhn and Thomas Moscibroda and Roger Wattenhofer What cannot be computed locally, IN *ACm PODC*, 2004, pages 300–309.

[24] Fabian Kuhn, Roger Wattenhofer, and Aaron Zollinger. Asymptotically optimal geometric mobile ad-hoc routing. In *ACM DIALM workshop*, 2002.

[25] Fabian Kuhn, Roger Wattenhofer, and Aaron Zollinger. Worst-case optimal and average-case efficient geometric ad-hoc routing. In *ACM MobiHoc*, 2003.

[26] Ning Li and Jennifer C. Hou, FLSS: a fault-tolerant topology cntrol algorithm for wireless networks, Proc. ACM Mobicom, September 2004

[27] Xiang-Yang Li, G. Calinescu, Peng-Jun Wan, and Yu Wang Localized Delaunay Triangulation with Applications in Wireless Ad Hoc Networks, IEEE Transactions on Parallel and Distributed Systems. October 2003 (Vol. 14, No. 10), pages 1035-1047.

[28] Xiang-Yang Li, Yu Wang, Peng-Jun Wan and Chi-Wei Yi Fault Tolerant Deployment and Topology Control for Wireless Ad Hoc Networks, In *ACM MobiHoc*, 2003. Journal version in WCMC, Volume 4, Issue 1, 2004.

[29] Li Li, Joseph Y. Halpern, Paramvir Bahl, Yi-Min Wang, and Roger Wattenhofer. Analysis of a cone-based distributed topology control algorithms for wireless multi-hop networks. In *ACM PODC*, 2001.

[30] Xiang-Yang Li. Approximate mst for UDG locally. In *COCOON*, 2003.

[31] Xiang-Yang Li, Peng-Jun Wan, Yu Wang, and Ophir Frieder. Sparse power efficient topology for wireless networks. In *IEEE Hawaii Int. Conf. on System Sciences (HICSS)*, 2002.

[32] Xiang-Yang Li, Yu Wang, Wen-Zhan Song, Peng-Jun Wan, and Ophir Frieder. Localized minimum spanning tree and its applications in wireless ad hoc networks. In *IEEE INFOCOM*, 2004.

[33] Weifa Liang. Constructing minimum-energy broadcast trees in wireless ad hoc networks In *3rd ACM MobiHoc*, 2002.

[34] Moni Naor and Larry Stockmeyer, What can be computed locally?, In *ACM STOC* 1993, pages 184–193

[35] Dragos Niculescu and Badri Nath. Ad hoc positioning system (APS). In *IEEE GLOBECOM (1)*, pages 2926–2931, 2001.

[36] V. Park and M. S. Corson. Temporally-ordered routing algorithm (tora) version 1 specification. IETF Internet Draft, November 2000. draft-ietf-manet-tora-spec-03.txt.

[37] Mathew Penrose. The longest edge of the random minimal spanning tree. *Annals of Applied Probability*, 7:340–361, 1997.

[38] C. E. Perkins, E. M. Belding-Royer, and S. Das. Ad hoc on demand distance vector (AODV) routing. IETF Internet Draft, March 2002. draft-ietf-manet-aodv-10.txt.

[39] C. E. Perkins and P. Bhagwat. Highly dynamic destination-sequenced distance-vector routing (DSDV) for mobile computers. *IEEE Communications Review*, pages 234–244, October 1994.

[40] Mahtab Seddigh, J. Solano Gonzalez, and I. Stojmenovic. Rng and internal node based broadcasting algorithms for wireless one-to-one networks. *ACM Mobile Computing and Communications Review*, 5(2):37–44, 2002.

[41] R. C. Shah and J. M. Rabaey. Energy aware routing for low energy ad hoc sensor networks. In *IEEE Wireless Communication and Networking Conference (WCNC) 2002*, pages 350–355, March 2002.

[42] Wen-Zhan Song, Yu Wang, Xiang-Yang Li, and Ophir Frieder. Localized algorithms for energy efficient topology in wireless ad hoc networks. In *ACM MobiHoc*, 2004.

[43] Godfried T. Toussaint. The relative neighborhood graph of a finite planar set. *Pattern Recognition*, 12(4):261–268, 1980.

[44] Peng-Jun Wan, G. Calinescu, Xiang-Yang Li, and Ophir Frieder. Minimum-energy broadcast routing in static ad hoc wireless networks. In *IEEE Infocom*, 2001.

[45] Peng-Jun Wan, G. Calinescu, Xiang-Yang Li, and Ophir Frieder. Minimum-energy broadcast routing in static ad hoc wireless networks. *ACM Wireless Networks*, Volume 8 , Issue 6 (November 2002), Pages: 607 - 617. Preliminary version appeared in IEEE INFOCOM 2000.

[46] Yu Wang and Xiang-Yang Li. Efficient construction of bounded degree and planar spanner for wireless networks. In *ACM DIALM-POMC Workshop*, 2003.

[47] L. Orecchia, A. Panconesi, C. Petrioli, A. Vitaletti. Localized techniques for broadcasting in wireless sensor networks, In *ACM DIALM-POMC*, 2004.

[48] WeiZhao Wang, Xiang-Yang Li, Kousha Moaveninejad, Yu Wang and Wen-Zhan Song, The Spanning Ratios of *beta*-Skeleton, In *Canadian Conference on Computational Geometry (CCCG)*, 2003.

[49] J. Wieselthier, G. Nguyen, and A. Ephremides. On the construction of energy-efficient broadcast and multicast trees in wireless networks. In *Proc. IEEE INFOCOM 2000*, pages 586–594, 2000.

[50] J. Wu and F. Dai. Broadcasting in ad hoc networks based on self-pruning. In *IEEE INFOCOM*, 2003.

[51] A. C.-C. Yao. On constructing minimum spanning trees in k-dimensional spaces and related problems. *SIAM Journal of Computing*, 11:721–736, 1982.

[52] F. Ye, H. Luo, J. Cheng, S. Lu, and L. Zhang. A two-tier data dissemination model for large-scale wireless sensor networks. In *ACM MobiCom*, 2002.

APPENDIX

Das *et al.* [10] proved that if a set of line segments E satisfies the isolation property, then $\omega(E) = O(1) \cdot \omega(SMT)$. Here SMT is the Steiner minimum tree over the end points of E, and total edge weight of SMT is no more than that of the minimum spanning tree. The isolation property is defined as follows. Let $c > 0$ be a constant and E be a set of edges in d-dimensional space, and let $e \in E$ be an edge of length l. If it is possible to place a *protecting disk* B of radius $c \cdot l$ with center on e and B does not intersect with any other edge, then edge e is said to be *isolated* [10]. If all the edges in E are isolated, then E is said to satisfy the *isolation property*. We define the *protecting disk* of a segment uv as $disk(o, \frac{\sqrt{35}}{36}\|uv\|)$, where o is the midpoint of segment uv. Obviously, we need all such disks do not intersect any edge except the one that defines it.

Theorem 6: The structure $LS\Theta GG$ is low-weighted.

Proof. We will prove this by showing that all edges E in $LS\Theta GG$ satisfy the isolation property. For the sake of contradiction, assume that E does not satisfy the isolation property. Assume there is one edge uv that is not isolated. Thus, there is

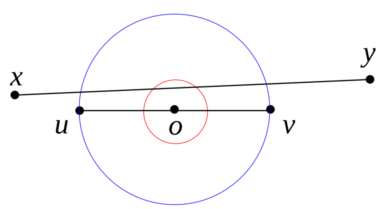


Figure 11: The hypothetical cases that an edge uv is not isolated. Here assume edge xy intersects its protecting $disk(o, \frac{\sqrt{35}}{36}\|uv\|)$.

an edge, say xy, that intersects the protecting disk of uv. Figure 11 illustrates the hypothetical situation: a link xy intersects the protecting disk of link uv, *i.e.*, $disk(o, \frac{\sqrt{35}}{36}\|uv\|)$. First notice that, both x and y can not locate inside $disk(o, \frac{1}{2}\|uv\|)$, otherwise the property of Gabriel graph is violated.

We further divide the hypothetical situation into two cases:

Case 1: $\|xy\| < \|uv\|$.

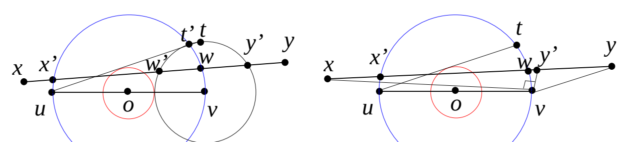
We will show that the link uv itself must have been removed by our algorithm, by proving that both $\|ux\|$ and $\|vy\|$ are no more than $\frac{1}{3}\|uv\|$ in the hypothetical situation. To prove this by inducing contradiction, w.l.o.g., we assume that $\|vy\| > \frac{1}{3}\|uv\|$.

Figure 12 (a) illustrates our proof that follows. The link xy intersects the $disk(o, \frac{1}{2}\|uv\|)$ with two points x' and w, and intersects the right half of $disk(v, \frac{1}{3}\|uv\|)$ with the point y'. Let t be a point on the top half of $disk(v, \frac{1}{3}\|uv\|)$ such that $\|ut\| = \|uv\|$. The segment ut intersects the $disk(o, \frac{1}{2}\|uv\|)$ with point t'. It is easy to verify that ut is the tangent line of protecting $disk(o, \frac{\sqrt{35}}{36}\|uv\|)$.

From the assumption $\|vy\| > \frac{1}{3}\|uv\|$, node y is out of the $disk(v, \frac{1}{3}\|uv\|)$. Hence, $\|xy\| > \|x'y'\|$. We continue to induce contradiction that $\|xy\| > \|uv\|$ by showing $\|x'y'\| > \|ut\| = \|uv\|$.

1. Obviously, $\|x'w\| > \|ut'\|$, because the chord $x'w$ of $disk(o, \frac{1}{2}\|uv\|)$ is closer the center o than the chord ut' (because $x'w$ intersects the protecting disk while ut is the tangent line).
2. Similarly, $\|wy'\| > \|tt'\|$, because $\|ww'\| > 2\|tt'\|$ (this is due to, in $disk(v, \frac{1}{3}\|uv\|)$, the chord ww' is closer to center v than line tt', and segment tt' is half of the chord overlapping tt' since vt' is perpendicular to ut' in $disk(o, \frac{1}{2}\|uv\|)$.) and $\|wy'\| > \frac{1}{2}\|ww'\|$ (due to in $disk(v, \frac{1}{3}\|uv\|)$, $\angle x'wv > \angle uwv = \frac{\pi}{2}$, hence $\|wy'\|$ is more than half of the chord ww'.).

Consequently, $\|xy\| > \|x'y'\| = \|x'w\| + \|wy'\| > \|ut'\| + \|tt'\| = \|ut\| = \|uv\|$, hence we get the contradiction. In other words, uv should have been deleted if $\|uv\| > \|xy\|$ and xy intersects the protecting disk of uv. The hypothetical case is fake.



(a) $\|xy\| < \|uv\|$ (b) $\|xy\| > \|uv\|$

Figure 12: Both cases are impossible.

Case 2: $\|xy\| > \|uv\|$.

We will show that xy will be deleted by Algorithm 2 by showing $\max(\|ux\|, \|vy\|) < \frac{1}{3}\|xy\|$ if xy intersects the protecting $disk(o, \frac{\sqrt{35}}{36}\|uv\|)$ of link uv. We prove it by inducing contradiction. W.l.o.g., assume that $\|vy\| > \frac{1}{3}\|xy\|$.

Figure 12 (b) illustrates our proof that follows. Here ut is a tangent line of the protecting disk, the link xy intersects the $disk(o, \frac{1}{2}\|uv\|)$ with two points x' and w. The segment $y'v$ is perpendicular to xv. Here point y' is on line xy.

Obviously, $\angle vxy < \angle vx'y$. And $\angle vx'y < \angle vut$ because the arc $\widehat{vw}$ is smaller than the arc $\widehat{vt}$. We have, $\|vy'\| = \|xy'\| \sin(\angle vxy) < \|xy'\| \sin(\angle vut) = \frac{\sqrt{35}}{18}\|xy'\| < \frac{1}{3}\|xy\|$. On the other hand, $\|vw\| < \|vt\| < \frac{1}{3}\|uv\| < \frac{1}{3}\|xy\|$. Hence, node y can not be on the left side of y', instead only possible on the right side since $\|vy\| > \frac{1}{3}\|xy\|$. Then, we have $\angle xvy > \frac{\pi}{2}$, *i.e.*, link xy cannot be in GG. Contradiction is induced.

Consequently, xy should have been deleted if $\|uv\| < \|xy\|$ and xy intersects the protecting disk of uv. The hypothetical case is also fake.

In summary, each link $uv \in LS\Theta GG$ satisfies the isolation property, that is to say, $LS\Theta GG$ is low-weighted. This finishes the proof. $\square$