Approximate MST for UDG locally

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Abstract. We consider a wireless network composed of a set of n wireless nodes distributed in a two dimensional plane. The signal sent by a node can be received by all nodes within its transmission region, which is a unit disk centered at this node. The nodes together define a unit disk graph (UDG) with edge uv iff $\|uv\| \leq 1$. We present the first localized method to construct a bounded degree planar connected structure for UDG whose total edge length is within a constant factor of the minimum spanning tree. The total communication cost of our method is $O(n)$, and every node only uses its two-hop information to construct such structure. We show that some two-hop information is necessary to construct any low-weighted structure. We also study the application of this structure in efficient broadcasting in wireless ad hoc networks. We prove that this structure uses only $O(1/n)$ of the total energy of the previously best-known light weighted structure RNG.

1 Introduction

We consider a wireless ad hoc network composed of n nodes distributed in a two-dimensional plane. Assume that all wireless nodes have distinctive identities and each static wireless node knows its position information either through a low-power Global Position System (GPS) receiver or through some other way. We assume that each wireless node has an omni-directional antenna and a single transmission of a node can be received by *any* node within its vicinity which, we assume, is a unit disk centered at this node. By one-hop broadcasting, each node u can gather the location information of all nodes within the transmission range of u. Consequently, all wireless nodes V together define a unit-disk graph UDG, which has an edge uv iff the Euclidean distance $\|uv\|$ is less than one unit. Throughout this paper, a *local broadcast* by a node means it sends the message to all nodes within its transmission range; a *global broadcast* by a node means it tries to send the message to all nodes in the network by the possible relaying of other nodes. Since the main communication cost in wireless networks is to send out the signal while the receiving cost of a message is neglected here, a protocol's message complexity is only measured by how many messages are sent out by all nodes.

In recent years, there are substantial amount of research on topology control for wireless ad hoc networks [1–4]. These algorithms are designed for different objectives: minimizing the maximum link length while maintaining the network connectivity [3]; bounding the node degree [2]; bounding the spanning ratio [2, 1]; constructing planar spanner locally [1]. Planar structures are used by several localized routing algorithms [5, 6]. We [7] recently also proposed the first localized algorithm to construct a bounded degree planar spanner. A structure is called *low weight* if its total edge length is within a constant factor of the minimum spanning tree (MST). However, no localized algorithm is known to construct a low-weighted structure. It was recently shown in [8] that a broadcasting based on MST consumes energy within a constant factor of the optimum.

The best distributed algorithm [9, 10] can compute MST in $O(n)$ rounds using $O(m+n\log n)$ communications for a general graph with m edges and n nodes. We can construct the minimum spanning tree of UDG in a distributed manner using $O(n\log n)$ messages. Unfortunately, even for wireless network modeled by a ring, the $O(n\log n)$ number of messages is still necessary for constructing MST of UDG. We present the first localized method to construct a bounded degree planar connected structure H whose total edge length is within a constant factor of MST. The total communication cost of our method is $O(n)$, and every node only uses its two-hop information to construct such structure. We also show that some two-hop information is necessary to construct any low-weighted structure. We also show the application of this structure in efficient broadcasting in wireless ad hoc networks.

Energy conservation is a critical issue in *ad hoc* wireless network for the node and network life, as the nodes are powered by batteries only. In the most common power-attenuation model, the power needed to support a link uv is $\|uv\|^{\beta}$, where β is a real constant between 2 and 5 dependent on the wireless transmission environment. Minimum-energy broadcast/multicast routing in ad hoc networking environment has been addressed in [11, 12]. Wan *et al.* [8] showed that the approximation ratios of MST-based approach is between 6 and 12. Unfortunately, MST cannot be constructed in a localized manner, i.e., each node cannot determine which edge is in the defined structure by purely using the information of the nodes within some constant hops. Relative neighborhood graph has been used for broadcasting in wireless ad hoc networks [13]. It is well-known that $MST \subseteq RNG$. The ratio of the weight of RNG over the weight of MST could be $O(n)$ for n points set [14]. As will show later, the total energy used by this approach could be about $O(n^{\beta})$ times optimum.

Notice that a structure with low-weight cannot guarantee that the broadcasting based on it consumes energy within a constant factor of the optimum. We show that the energy consumption using the structure H is within $O(n^{\beta-1})$ of the optimum, i.e., $\omega_{\beta}(H) = O(n^{\beta-1}) \cdot \omega_{\beta}(MST)$. This improves the previously known "lightest" structure RNG by $O(n)$ factor since in the worst case $\omega_{\beta}(RNG) = \Theta(n^{\beta}) \cdot \omega_{\beta}(MST)$.

The remainder of the paper is organized as follows. We present our efficient localized method constructing a bounded degree planar structure with low weight in Section 2. In Section 3, we discuss its applications in broadcasting, and find that it saves considerable energy consumption compared with that based on RNG. We conclude our paper in Section 4 with the discussion of possible future works.

2 Low Weight Topology

Let $\|xy\|$ denote the Euclidean distance between two points x and y. A disk centered at a point x with radius r is denoted by $disk(x, r)$. Let $lune(u, v)$ defined by two points u and v be the intersection of two disks with radius $\|uv\|$ and centered at u and v respectively, i.e., $lune(u, v) = disk(u, \|uv\|) \cap disk(v, \|uv\|)$. The *relative neighbor-*

hood graph [15], denoted by RNG(V), consists of all edges uv such that the *interior* of $lune(u, v)$ contains no point $w \in V$. Notice here if only the boundary of $lune(u, v)$ contains a point from V, edge uv is still included in RNG. Given a geometry graph G over a set of points, let $\omega(G)$ be the total length of the edges in G. More specifically, if the weight of an edge uv is defined as $\|uv\|^b$, then let $\omega_b(G)$ be the total weight of the weighted edges in G, i.e., $\omega_b(G) = \sum_{uv \in G} \|uv\|^b$. When $b = 1$, b is often omitted from the notations. It is known that $MST \subseteq RNG$.

2.1 Modified RNG

Our low-weight structure is based on a modified relative neighborhood graph. Notice that, traditionally, the relative neighborhood graph will always select an edge uv even if there is some node on the boundary of $lune(u, v)$. Thus, RNG may have unbounded node degree, e.g., considering $n - 1$ points equally distributed on the circle centered at the nth point v, the degree of v is $n - 1$.

The modified *relative neighborhood graph* consists of all edges uv such that (1) the *interior* of $lune(u, v)$ contains no point $w \in V$ and, (2) there is no point $w \in V$ with $ID(w) < ID(v)$ on the boundary of $lune(u, v)$ and $\|wv\| < \|uv\|$, and (3) there is no point $w \in V$ with $ID(w) < ID(u)$ on the boundary of $lune(u, v)$ and $\|wu\| < \|uv\|$, and (4) there is no point $w \in V$ on the boundary of $lune(u, v)$ with $ID(w) < ID(u)$, $ID(w) < ID(v)$, and $\|wu\| = \|uv\|$. See Figure 1 for an illustration when an edge uv is *not* included in the modified relative neighborhood graph. We denote such structure by RNG' hereafter. Obviously, RNG' is a subgraph of traditional RNG. We prove that RNG' has a maximum node degree 6 and still contains a MST as a subgraph. The proof of the following lemma is omitted due to space limitation.

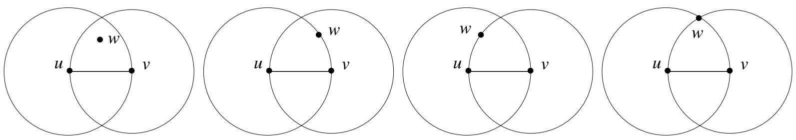

Fig. 1. Four cases when edges are not in the modified RNG.

Lemma 1. *The maximum node degree in graph RNG' is at most* 6.

Lemma 2. *The graph RNG' contains a Euclidean MST as a subgraph.*

PROOF. One way to construct MST is to add edges in the order of their lengths to the MST if it does not create a cycle with previously added edges. We break the tie as follows. We label an edge uv by $(\|uv\|, \max(ID(u), ID(v)), \min(ID(u), ID(v)))$, and an edge uv is ordered before an edge xy if the lexicographic order of the label of uv is less than that of xy. Let T be the MST constructed using the above edge ordering. We can show that $T \subseteq RNG'$. The detail is omitted due to space limitation. $\sqcup\!\!\!\!\sqcap$

Obviously, graph RNG' still can be constructed using n messages. Each node first locally broadcasts its location and ID to its one-hop neighbors. Then every node decides which edge to keep solely based on the one-hop neighbors' location information collected. Since the definition is still symmetric, the edges constructed by different nodes are consistent, i.e., an edge uv is kept by a node u iff it is also kept by node v. The computational cost of a node u is still $O(d \log d)$, where d is its degree in UDG. A simple edge by edge testing method has time complexity $O(d^2)$.

Although graph RNG' has possibly less edges than RNG, its total edge weight could still be arbitrarily large compared with the MST. Figure 2 (a) illustrates an example where $\omega(RNG')/\omega(MST) = O(n)$ for a set of n points. Here $n/2$ points are equally distributed with separation $\epsilon \leq 2/n$ on two parallel vertical segments with distance 1 respectively. Obviously, all edges forming RNG' have total weight $n/2 + (n-2)\epsilon$ and the MST has total weight $1 + (n-2)\epsilon$. On the other hand, we have

Lemma 3. *For any sparse graph G with $O(n)$ edges, containing MST as subgraph, $\omega_b(G) = O(n^b) \cdot \omega_b(MST)$ for $b \geq 1$, and it has length spanning ratio at most $O(n)$.*

PROOF. For any edge $uv \in G$, if $uv \in MST$, then $\|uv\|^b < \omega_b(MST)$. If $uv \notin MST$, then there is a path in MST with edges not longer than uv connecting u and v. Let a_j, $1 \leq j \leq k \leq n$ be the lengths of these edges. Then $\|uv\| < \sum_{1 \leq j \leq k} a_j$. Thus, $\|uv\|^b < (\sum_{1 \leq j \leq k} a_j)^b \leq n^{b-1} \sum_{1 \leq j \leq k} a_j^b \leq n^{b-1} \cdot \omega_b(MST)$. Consequently, $\omega_b(G) = O(n^b) \cdot \omega_b(MST)$ since G has only $O(n)$ edges. Similar proof can show that G has length spanning ratio at most $O(n)$. □

2.2 Bound the Weight

In this section, we give a communication efficient method to construct a sparse topology from RNG' whose total edge weight is within a constant factor of $\omega(MST)$. Previously no localized method is known to construct a structure with weight $O(\omega(MST))$.

We first show by example that it is *impossible* to construct a low-weighted structure using only one hop neighbor information. Assume that there is such algorithm. Consider a set of points illustrated by Figure 2 (a). Let's see what this hypothetical algorithm will do to this point set. Since it uses only one-hop information, at every node, the algorithm only knows that there is a sequence of nodes evenly distributed with small separation, and another node which is one-unit away from current node. Since the algorithm has the same (or almost same) information at each node, the algorithm cannot decide whether to keep the long edge. If it keeps the long edge, then the total weight of the final structure is $O(n \cdot \omega(MST))$. If it discards the long edge, however, it may disconnect the graph since the nodes known by the algorithm at one node may be the whole network. See Figure 2 (b) for an illustration. We then give the first localized algorithm that constructs a low-weighted structure using only some two hops information.

Algorithm 1 *Construct Low Weight Structure*

1. All nodes together construct the graph RNG' in a localized manner.

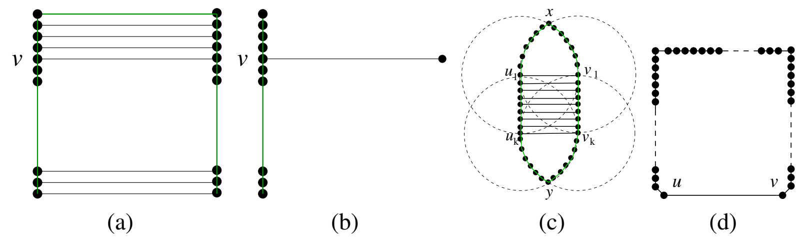

Fig. 2. The hypothetical algorithm cannot distinguish two cases (a) and (b) here. (c): Broadcasting based on RNG is expensive. (d) $\omega_\beta(H) = O(n^{\beta-1}) \cdot \omega_\beta(MST)$.

2. Each node u locally broadcasts its incident edges in RNG' to its one-hop neighbors. Node u listens to the messages from its one-hop neighbors.
3. Assume node u received a message informing existence of edge xy from its neighbor x. For each edge uv in RNG', if uv is the longest among xy, ux, and vy, node u removes edge uv. Ties are broken by the label of the edges. Here assume that $uvyx$ is the convex hull of u, v, x, and y.

Let H be the final structure formed by all remaining edges in RNG', and we call it low weighted modified relative neighborhood graph. Obviously, if an edge uv is kept by node u, then it is also kept by node v. To study the total weight of this structure, we will show that the edges in H satisfies the *isolation property* [16], which is defined as follows. Let $c > 0$ be a constant. Let E be a set of edges in d-dimensional space, and let $e \in E$ be an edge of weight l. If it is possible to place a hyper-cylinder B of radius and height $c \cdot l$ each, such that the axis of B is a subedge of e and B does not intersect with any other edge, i.e., $B \cap (E - \{e\}) = \phi$, then edge e is said to be *isolated* [16]. If all the edges in E are isolated, then E is said to satisfy the *isolation property*. The following theorem is proved by Das *et al.* [16].

Theorem 1. *[16] If a set of line segments E in d-dimensional space satisfies the isolation property, then $\omega(E) = O(1) \cdot \omega(SMT)$.*

Here SMT is the Steiner minimum tree over the end points of E. Obviously, total edge weight of SMT is no more than that of the minimum spanning tree. Generally, $\omega(MST) = O(\omega(SMT))$ for a set of points in Euclidean space. It is also known [16] that, in the definition of the isolation property, we can replace the hyper-cylinder by a hypersphere, a hypercube etc., without affecting the correctness of the above theorem. We will use a disk and call it *protecting disk*. Specifically, the protecting disk of a segment uv is $disk(p, \frac{\sqrt{3}}{4}\|uv\|)$, where p is the midpoint of segment uv. Obviously, we need all such disks do not intersect any edge except the one that defines it.

We first partition the edges of H into at most 8 groups such that the edges in each group satisfy the isolation property. Notice, given any node u, any cone apexed at u with angle less than $\pi/3$ will contain at most one edge of H incident on u since $H \subseteq RNG'$. Thus, we partition the region surrounded the origin by 8 equal-sized cones, say $\mathbb{C}_1$, $\mathbb{C}_2$, $\cdots$, $\mathbb{C}_8$ (the cone is half-open and half-close). The cones at different nodes are just a simple shifting of cones from the origin. Let E_i be the set of edges at cone C_i (one end-point is the apex of the cone and the other end-point is inside the cone).

Lemma 4. *No two edges in E_i share an end-point.*

PROOF. Assume that there are two edges xu and yu share a common node u, then obviously, these two edges cannot be from the cone apexed at node u; Clearly, angle $\angle xuy \leq 2\pi/8$. However, we already showed that there are no two edges incident on u form an angle less than $\pi/3$. This finishes the proof. ◻

Theorem 2. *The total edge weight of H is $O(\omega(MST))$.*

PROOF. We basically just show that the edges in E_i satisfy the isolation property, for $1 \leq i \leq 8$. For the sake of contradiction, assume that E_i does not satisfy the isolation property. Consider any edge uv from E_i and assume that it is not isolated. Thus, there is an edge, say xy, intersects the protecting disk of uv. There are four different cases: Case (a): $x \in disk(u, ||uv||)$ and $y \in disk(v, ||uv||)$; Case (b): $x \in disk(u, ||uv||)$ and $y \notin disk(v, ||uv||)$; Case (c): $x \notin disk(u, ||uv||)$ and $y \in disk(v, ||uv||)$; Case (d): $x \notin disk(u, ||uv||)$ and $y \notin disk(v, ||uv||)$. These four cases are illustrated by Figure 3. We will show that none of these four cases is possible.

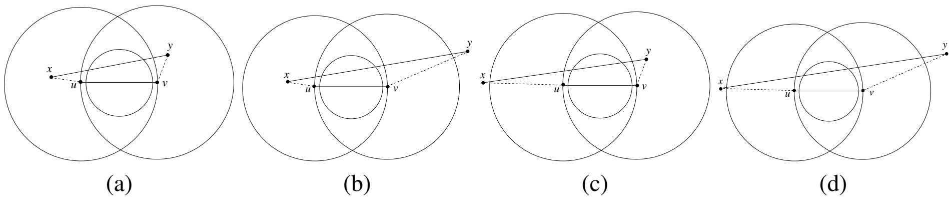

Fig. 3. Four cases that an edge uv is not isolated. Assume edge xy intersects the protecting disk.

For the first case, since x is in $disk(u, ||uv||)$ and y is in $disk(v, ||uv||)$, we know that xu and yv are both shorter than uv. Here, xu and yv need not be in the structure E_i. Thus, either uv or xy is the longest edge among uv, xy, xu and yv. Consequently, our algorithm will remove either uv or xy (whichever is longer).

For the remaining three cases, we will show that edge xy is the longest of these four edges. First of all, nodes x and y cannot be on the different side of the line passing through nodes u and v. Assume that they do, and x is below the line uv. Assume that x is outside of the disk centered at u with radius $||uv||$ since one of the x and y is outside of the corresponding disk. See Figure 4 for an illustration. We first show that $\angle yxu < \pi/3$. Let q be the intersection point of segment xy with line uv. Let p be the corner point of the lune $lune(u, v)$ that is on the same side of uv as y. Obviously, $\angle yxu < \angle yqu < \angle pqu < \angle puv = \pi/3$. We then show that $||xy|| > ||xu||$. Let z be the intersection point of xy with the boundary of $lune(u, v)$ and closer to u than v. Obviously, $\angle xuz > \pi/2$, thus, $||xy|| > ||xz|| > ||xu||$. Consequently, point u is inside the lune defined by points x and y, which is a contradiction to the fact that $xy \in RNG'$.

We then prove that the Case (b) is impossible. Assume that y is outside of disk centered at v with radius $||uv||$. See Figure 4 (2) for an illustration of the proof that

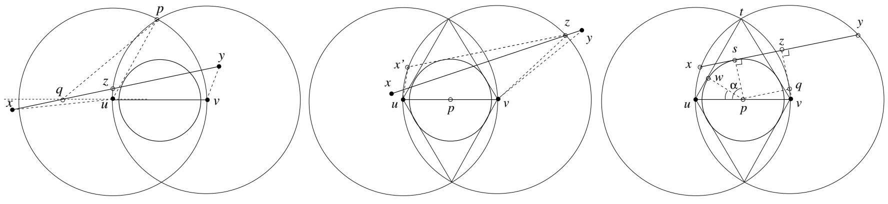

Fig. 4. (1) Node x is below uv and y is above. (2) Case (b) is impossible. (3) Edge xy is longest.

follows. Let z be the intersection point of xy with $disk(v, \|uv\|)$ that is closer to y. Let x' be the point on the disk $disk(v, \|uv\|)$ such that segment zx' is tangent to the protecting disk of segment uv. Obviously, $\angle ux'z > \pi/2$. Then $\|zx\| > \|zx'\|$. We can show that $\|zx'\|$ is at least $\|zv\|$ (the proof is presented in next paragraph). Then

$$\|yx\| > \|yz\| + \|zx'\| > \|yv\| - \|zv\| + \|zx'\| > \|yv\|$$

Then xy is the longest segment of the convex hull $xyvu$ since $\|xu\| \leq \|uv\| \leq \|vy\|$. This is a contradiction since our algorithm will remove edge xy. Notice here edge xy is the longest edge implies that node u is a neighbor of x and node v is a neighbor of y. Thus both node x and node y will know the existence of edge uv, and thus will remove edge xy according to our algorithm.

Figure 4 (3) illustrates the proof of $\|zx'\| \geq \|zv\|$ that follows. Consider any chord xy tangent on the protecting disk for uv. We then prove $\|xy\| \geq \|yv\| = \|uv\|$. Let z be the midpoint of xy, i.e., vz is perpendicular to xy. To make xy shorter, segment vz must be as long as possible. Let p be the midpoint of uv and s be the point on xy such that segment ps is perpendicular to segment xy. Then clearly, $\|vz\| = \|ps\| + \|pv\| \cdot \cos(\angle ups)$. Thus, xy is minimized when angle $\angle ups$ is minimized. However, $\angle ups > \angle upw$ since x and y are all above the line uv. Here w is the only intersecting point of chord ut with the protecting disk $disk(p, \frac{\sqrt{3}}{4}\|uv\|)$. It is easy to show $\|ut\| = \|tv\| = \|uv\|$. Thus, the minimum length of segment xy is $\|uv\|$ when $\angle ups = \angle upw$.

The proof of Case (c) is exactly the same as that for Case (b). For Case (d), same to the proof of Case (b), we know that $\|xu\| < \|xy\|$ and $\|vy\| < \|xy\|$. Then edge xy is also the longest edge of the convex hull $xyvu$. This is a contradiction since our algorithm will remove edge xy (nodes x and y will be informed by u and v respectively of the existence of edge uv since $\|xu\| < 1$ and $\|vy\| < 1$). This finishes the proof. ◻

Lemma 5. *The constructed topology H still contains a MST as a subgraph.*

PROOF. Consider the minimum spanning tree T constructed in the proof of Lemma 2. We will prove that $T \subseteq H$ by induction on the order of the edges added to the minimum spanning tree T. For the edge with the smallest order, it is clearly still in H. Assume that the first $k-1$ edges added to T are still in H. Consider the kth edge, say uv, added to T. If uv is not in H, there must have two points x and y such that edge uv has the largest lexicographical label among edges on the convex hull $uvyx$.

Notice that for RNG', it is easy to show by induction that, for any two points p and q, there is a path in RNG' connecting p and q, whose edges have label less than that of pq. For any edge in this path, if it is not in T, then by definition of T, we know that there is another path with edges in T to connect the two endpoints of this edge. Thus, for any two points p and q, there is a path in T connecting p and q, whose edges have label less than that of pq.

Consequently, for points u and v, there is a path in T connecting them using edges with label lexicographically less than uv. This is a contradiction to the fact that uv is also in the minimum spanning tree T. This finishes the proof. ◻

3 Application in Broadcasting

Minimum-energy broadcast/multicast routing in a simple ad hoc networking environment has been addressed in [11, 12]. Any broadcast routing is viewed as an arborescence (a directed tree) T, rooted at the source node of the broadcasting, that spans all nodes. Let $f_T(p)$ denote the transmission power of the node p required by T. For any leaf node p of T, $f_T(p) = 0$. For any internal node p of T, $f_T(p) = \max_{pq \in T} \|pq\|^{\beta}$, i.e., the β-th power of the longest distance between p and its children in T. The total energy required by T is $\sum_{p \in V} f_T(p)$. It is known [17] that the minimum-energy broadcast routing problem cannot be solved in polynomial time if $P \neq NP$. Wan *et al.* [8] showed that the approximation ratio of MST based approach is between 6 and 12.

Lemma 6. *For any point set V in the plane, the total energy required by any broadcasting among V is at least $\omega_\beta(MST)/C_{mst}$, where $6 \leq C_{mst} \leq 12$ is a constant related to the geometry minimum spanning tree.*

RNG has been used for broadcasting in wireless ad hoc networks [13]. Obviously, the ratio of the weight in RNG over the weight of MST could be $O(n)$ for n points set [14]. Figure 2 (c) illustrates an example that the total energy used by broadcasting on RNG could be about $O(n^\beta)$ times of the minimum-energy used by an optimum method. Here the n nodes are evenly distributed on the arc xu_1, segment u_1u_k, arc u_ky, arc yv_k, segment v_kv_1, and arc v_1x. Here four nodes u_1, u_k, v_1, and v_k form a unit square. It is not difficult to show that $\omega(MST) = \Theta(1)$ and $\omega_\beta(MST) = \Theta(1/n^{\beta-1})$, while $\omega(RNG) = \Theta(n)$ and $\omega_\beta(RNG) = \Theta(n)$. Together with Lemma 3, we know that in the worst case, $\omega_\beta(RNG') = \Theta(n^\beta) \cdot \omega_\beta(MST)$.

Notice that although the structure H has total edge weight $\omega(H) \leq c \cdot \omega(MST)$ for some constant c, it does not guarantee that $\omega_\beta(H)$ is within a constant factor of $\omega_\beta(MST)$ for $\beta > 1$. Figure 2 (d) illustrates such an example. Here the segment uv has length 1. The other $n-2$ nodes are evenly distributed along the three segments of a square (with side length $1+\epsilon$) such that the lines drawn in Figure 2 is indeed the graph RNG'. It is not difficult to show that $H = RNG'$. Obviously, $\omega_\beta(H) = O(1)$ and $\omega_\beta(MST) = O(1/n^{\beta-1})$ for any $\beta > 1$. On the other hand, we have

Lemma 7. $\omega_\beta(H) \leq O(n^{\beta-1}) \cdot \omega_\beta(MST)$.

PROOF. Assume that $\omega(H) \leq c \cdot \omega(MST)$ for a constant c. Let a_i, $1 \leq i \leq k$ be the edge lengths of H, and b_i, $1 \leq i \leq n-1$ be the edge lengths of MST. Here $k = O(n)$ is the number of edges in H. Then $\sum_{1\leq i\leq k} a_i^\beta \leq (\sum_{1\leq i\leq k} a_i)^\beta \leq c^\beta \cdot (\sum_{1\leq i\leq n-1} b_i)^\beta \leq c^\beta \cdot n^{\beta-1} \cdot \sum_{1\leq i\leq n-1} b_i^\beta$.This finishes the proof. ◻

Consequently, we know that in the worst case, $\omega_\beta(H) = \Theta(n^{\beta-1})\cdot\omega_\beta(MST)$. Figure 2 (d) shows that, to get a structure with weight $O(\omega_\beta(MST))$, we have to construct a MST for that example. Notice that the worst case communication cost to build a MST is $O(n \log n)$ under the broadcast communication model. It seems that the we may have to spend $O(n \log n)$ communications to build a structure with weight $O(\omega_\beta(MST))$. However, this worst case may not happen at all: for the configurations of nodes that need $O(n \log n)$ communications to build MST, the structure built by our method may be good enough; on the other hand, for the example that our algorithm does not perform well, we may find an efficient way to build a MST using $o(n \log n)$ messages. We leave it as an open problem whether we can construct a structure G whose weight $\omega_\beta(G)$ is $O(\omega_\beta(MST))$ using only $o(n \log n)$ messages, or even $O(n)$ messages. Here each message has $O(\log n)$ bits always.

4 Conclusion

We consider a wireless network composed of n a set of wireless nodes distributed in a two dimensional plane. We presented the first localized method to construct a bounded degree planar connected structure H with $\omega(H) = O(1) \cdot \omega(MST)$. The total communication cost of our method is $O(n)$, and every node only uses its two-hop information to construct such structure. We showed that some two-hop information is necessary to construct any low-weighted structure. We also studied the application of this structure in efficient broadcasting in wireless ad hoc networks. We showed that the energy consumption using this structure is within $O(n^{\beta-1})$ of the optimum, i.e., $\omega_\beta(H) = O(n^{\beta-1}) \cdot \omega_\beta(MST)$ for any $\beta \geq 1$. This improves the previously known "lightest" structure RNG by $O(n)$ factor since $\omega(RNG) = \Theta(n) \cdot \omega(MST)$ and $\omega_\beta(RNG) = O(n^\beta) \cdot \omega_\beta(MST)$.

On one aspect, a structure with low-weight does not guarantee that it approximates the optimum broadcasting structure in terms of the total energy consumption. On the other hand, a structure for broadcasting whose total energy consumption is within a constant factor of optimum does not guarantee that it is low-weight. We can show that its total edge length is within $O(\sqrt{n})$ of $\omega(MST)$ for a n-nodes network. Considering this "non-relevance" of the low-weight structure and the optimum broadcasting structure, it remains open how to construct a topology good for broadcasting.

The constructed structure is planar, and has bounded degree, low-weight. We [18] recently gave an $O(n \log n)$-time centralized algorithm constructing a bounded degree, planar, and low-weighted spanner. However, we cannot make that a distributed algorithm using $O(n)$ communications without sacrificing the spanner property. On the other hand, we [7] showed how to construct a planar spanner with bounded degree in a localized manner (using $O(n)$ messages) for unit disk graph. However, the constructed structure does not seem to have low-weight. It remains open how to construct a bounded degree, planar, and *low-weighted spanner* in a distributed manner using only $O(n)$ communications under the local broadcasting communication model.

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