

1 Objectives

Unification is a third major topic that will appear many times in this course. It is used in languages such as OCaml and Prolog, and also in theoretical discussions.

- Be able to describe the problem of unification.
- Be able to solve a unification problem.
- Know how to use unification to implement pattern matching.
- Know how to use unification to check types of functions.

2 Unification Operations

Start with a unification problem $S = \{s_1 =? t_1, s_2 =? t_2, \dots\}$ and apply the following transformations as necessary:

Delete A trivial equation $t =? t$ can be deleted.

Decompose An equation $f(\overline{t_n}) =? f(\overline{u_n})$ can be replaced by the set $\{t_1 =? u_1, \dots, t_n =? u_n\}$

Orient An equation $t =? x$ can be replaced by $x =? t$ if x is a variable and t is not.

Eliminate an equation $x =? t$ can be used to substitute all occurrences of x in the remainder of S .

3 Problems

Try the following problems. In a few minutes the instructor will go over the solutions. Feel free to work with the person next to you!

Try to unify the following:

- $\{f(\alpha, y) = f(x, \beta)\}$
- $\{f(\alpha, y) = f(x, \alpha)\}$
- $\{f(\alpha, \beta) = \gamma, \quad \gamma = f(x, \delta), \quad \beta = g(y)\}$
- $\{f(\alpha, \beta) = \gamma, \quad \gamma = f(x, \delta)\}$