

User-Defined Datatypes

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Example: Complex Numbers

§1 Record Types

- Complex numbers have the form $a + bi$, where $i \equiv \sqrt{-1}$
- Addition: $(a + bi) + (c + di) = (a + c) + (b + d)i$
- Multiplication: $(a + bi) \times (c + di) = ac - bd + (ad + bc)i$

```
1 # let cadd (a,b) (c,d) = a +. c, b +. d
2 # let cmul (a,b) (c,d) = a *. c -. b *. d,
3                       a *. d +. b *. c
4 # cmul (3.0,4.0) (1.0,4.4);;
5 - : float * float = -14.6, 17.2
```

We could use tuples to represent complex numbers, like this. (What are the types of these functions?) Why might this be a bad idea?

Objectives

§0 Objectives

Even more powerful than the idea of recursion is the idea of abstraction—which is a lot easier to implement if you can make your own types. Your goal after this lecture is to...

- Be able to describe the record type and the disjoint type.
- Be able to give an example of how each one is used.
- Be able to draw a memory diagram that is the result of allocating a variable of a disjoint type.
- Be able to use the `match/with` syntax to deconstruct a disjoint type.
- Be able to describe how data-structures in a functional language “recycle” data from previous versions of the structure when a modification has been made.

Record Type Definitions

§1 Record Types

type *name* = { *name* : *type* [*;* *name* : *type* ...] }

Name Labels

```
1 # type complex = { re : float; im : float };;
2 type complex = { re : float; im : float; }
3 # let cadd x y = { re = x.re +. y.re;
4                  im = x.im +. y.im };;
5 val cadd : complex -> complex -> complex = <fun>
```

- Fields are accessed with dot notation.
- Note that *field names must be unique*.

Activity**§1 Record Types**

1. Write a function to multiply complex numbers. Use the type definition we had before.
2. What would the type definition for a phone book entry look like? Assume we want to store a name, address, and phone number, all as strings.

Answer**§1 Record Types**

1. Write a function to multiply complex numbers. Use the type definition we had before.

```
1 let mult a b = { re = a.re * b.re - a.im * b.im;
2               im = a.re * b.im + b.re * a.im; }
```

2. What would the type definition for a phone book entry look like? Assume we want to store a name, address, and phone number, all as strings.

```
1 type address = {name : string; address : string;
2               phone : string };;
```

Simple Type Definitions**§2 Disjoint Types**

type *name* = *Name* [of type] [| *Name* [of type] ...]

Name

Constructors

Arguments

- Note: Constructor names must be capitalized.
- Constructor names also must be unique.

```
1 # type contest = Rock | Scissors | Paper
2 # type velocity = MeterPerSecond of float
3               | FeetPerSecond of float;;
```

Example of contest and velocity**§2 Disjoint Types**

```
1 # let winner a b =
2     match a,b with
3     | Rock,Scissors | Scissors,Paper | Paper,Rock
4     -> "Player 1"
5     | Rock,Paper | Scissors,Rock | Paper,Scissors
6     -> "Player 2"
7     | _ -> "Tie";;
8 val winner : contest -> contest -> string = <fun>
9 # let thrust vel = If only NASA had used OCaml!
10    match vel with
11    | FeetPerSecond x -> x /. 3.28
12    | MetersPerSecond x -> x;;
13 val thrust : velocity -> float = <fun>
```

```

1 # type foo = Foo of int;;
2 # let showfoo (Foo n) = n;;

1 # #use "foo.ml";; This loads the file foo.ml
2 # let t1 = Foo 5;;
3 # showfoo t1;;
4 - : int = 5
5 # #use "foo.ml";; At this point you reload....
6 # showfoo t1;;
7 Characters 8-10:
8 This expression has type foo but is here used
9 with type foo

```

Why did this happen?

```

1 # type mylist = Cons of int * mylist
2   | Nil
3 # let rec mklist lst =
4   match lst with
5   | [] -> Nil
6   | x::xs -> Cons (x, mklist xs);;
7 # let l1 = mklist [2;3;4];;
8 val l1 : mylist = Cons (2, Cons (3, Cons (4, Nil)))

```

- A recursive type without a recursive case is not really recursive.
- A recursive type without a base case is useless. Unless you're using Haskell, which supports infinite data-structures. But we're not going to talk about that here.

- When a type constructor is invoked, it causes memory to be allocated.
 - Writing an integer
 - Writing [] or ::
 - Using :: or Cons
- Writing down a variable does not cause memory to be allocated.

```

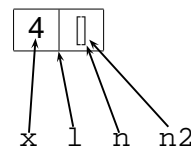
1 # let x = 4;; allocates 4
2 # let n = [];; allocates empty list
3 # let n2 = n;; does NOT allocate memory
4 # let l = x::n;; A cons cell is allocated,
5   but not the 4 or the empty list

```

```

1 # let x = 4;; allocates 4
2 # let n = [];; allocates empty list
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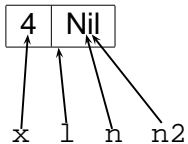
```



```

1 # let x = 4;;   allocates 4
2 # let n = Nil;; allocates Nil
3 # let n2 = n;;  does NOT allocate memory
4 # let l = Cons (x,n);; A cons cell is allocated,
5   but not the 4 or a Nil

```



How would you write length and inclist for this type?

```

1 # let rec length lst =
2   match lst with
3   | Nil -> 0
4   | Cons(x,xs) -> 1 + length xs;;
5 # length l1
6 - : int = 3
7 # let rec inclist lst =
8   match lst with
9   | Nil -> Nil
10  | Cons(x,xs) -> Cons(x+1,inclist xs);;
11 # inclist l1;;
12 - : mylist = Cons (3, Cons (4, Cons (5, Nil)))

```

```

1 # let rec reverse lst =
2   let rec aux lst acc =
3     match lst with
4     | Nil -> acc
5     | Cons(x,xs) -> aux xs (Cons(x,acc)) in
6   aux lst Nil;;
7 # reverse l1;;
8 - : mylist = Cons (4, Cons (3, Cons (2, Nil)))

```

OCaml supports parametric polymorphism, like templates in C++ or generics in Java.

```

1 # type 'a mylist = Cons of 'a * 'a mylist
2   | Nil
3 # let rec length xx =
4   match xx with
5   | Nil -> 0
6   | Cons (_,xs) -> 1 + length xs;;
7 val length : 'a mylist -> int = <fun>
8 # length (Cons(3,Cons(4,Nil)));;
9 - : int = 2
10 # length (Cons("Hi",Cons("There",Nil)));;
11 - : int = 2

```

1. Write the type for a binary search tree. In this version, let a “node” be something that has data, and a “leaf” does not have any data. Use parametric polymorphism.
2. Write `find` and `add`.

Let's take these one at a time.

```

1 let rec find a xx =
2   match xx with
3   | Node (x,l,r) when x=a -> true
4   | Node (x,l,r) -> if x<a then find a l
5                       else find a r
6   | Leaf -> false

```

Now write `add`. If the element is already added, return the value unchanged.

Write the type for a binary search tree.

```

1 type 'a bst = Node of 'a * 'a BST * 'a BST
2             | Leaf

```

Now write `find`.

```

1 let rec add a xx =
2   match xx with
3   | Node (x,l,r) when x=a -> xx
4   | Node (x,l,r) -> if x<a then Node(x, add a l, r)
5                       else Node(x, l, add a r)
6   | Leaf -> Node(a,Leaf,Leaf)

```

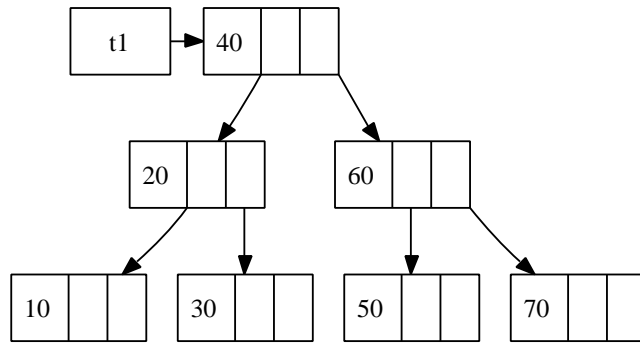
What will be the result of

```

1 let t1 = add 10 (add 30 (add 50 (add 70
2   (add 20 (add 60 (Node 40 Leaf Leaf))))));;

```

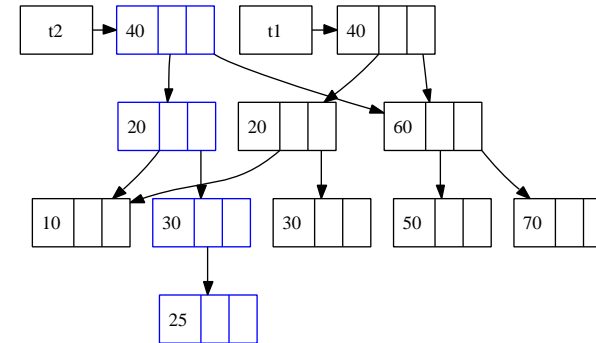
Before:



Now execute:

```
1 let t2 = add 25 t1;;
```

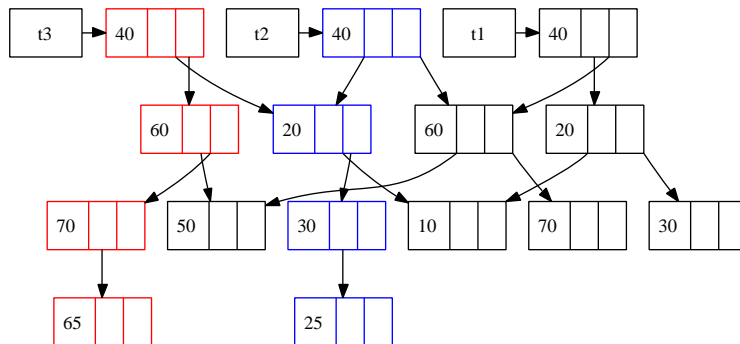
Before:



Now execute:

```
1 let t3 = add 65 t2;;
```

Before:



- This technique is called *functional-style updating*.
- Advantage: multiple versions of the data-structure can exist. This allows multiple versions, and consistent multiprocessing. Used to implement multi-level undo.
- Disadvantage: it takes more memory.

```
1 # type 'a option = Some of 'a | None;;
2 # let rec getItem key lst =
3     match lst with
4     | [] -> None      note the type variables!
5     | (k,v)::xs -> if key = k then Some v
6                     else getItem key xs;;
7 # getItem 3 [2,"french hens"; 3,"turtle doves"];
8 - : string option = Some "turtle doves"
9 # getItem 5 [2,"french hens"; 3,"turtle doves"];
10 - : string option = None
```

What options would you have for this function if you did *not* have an option type?