1 Objectives

By the end of lecture, you should know

- the different parts of a natural rule
- how to structure a proof-tree
- what the word "semantics" means.

2 Proof Rules

Integers

$$\frac{}{n \Downarrow n}$$

Parenthesis

$$\frac{e \Downarrow V}{(e) \Downarrow V}$$

Booleans

$$\frac{}{\texttt{true} \Downarrow true} \qquad \frac{}{\texttt{false} \Downarrow false}$$

if

$$\frac{e_1 \Downarrow true \quad e_2 \Downarrow V}{\texttt{if } e_1 \texttt{ then } e_2 \texttt{ else } e_3 \Downarrow V}$$

$$\frac{e_1 \Downarrow false \quad e_3 \Downarrow V}{\texttt{if } e_1 \texttt{ then } e_2 \texttt{ else } e_3 \Downarrow V}$$

and

$$\frac{e_1 \Downarrow true \quad e_2 \Downarrow v_2}{e_1 \texttt{ \&\& } e_2 \Downarrow v_2} \qquad \frac{e_1 \Downarrow false}{e_1 \texttt{ \&\& } e_2 \Downarrow false}$$

or

$$\frac{e_1 \Downarrow false \quad e_2 \Downarrow v_2}{e_1 \texttt{ || } e_2 \Downarrow v_2} \qquad \frac{e_1 \Downarrow true}{e_1 \texttt{ || } e_2 \Downarrow true}$$

Addition

$$\frac{e_1 \Downarrow v_1 \quad e_2 \Downarrow v_2}{e_1 + e_2 \Downarrow v_1 + v_2}$$

General Operators

$$\frac{e_1 \Downarrow v_1 \quad e_2 \Downarrow v_2}{e_1 \oplus e_2 \Downarrow v_1 \oplus v_2}$$

functions

$$\frac{}{\texttt{fun x -> } e_1 \Downarrow \texttt{fun x -> } e_1}$$

application

$$\frac{e_1 \Downarrow \texttt{fun x -> } e_2 \quad [e_3/\texttt{x}]e_2 \Downarrow V}{e_1 \ e_3 \Downarrow V}$$

let

$$\frac{[U/\texttt{x}]e_2 \Downarrow V \quad e_3 \Downarrow U}{\texttt{let x} = e_3 \texttt{ in } e_2 \Downarrow V}$$

3 Problems

Try the following problems. In a few minutes the instructor will go over the solutions. Feel free to work with the person next to you! You'll probably want to use the back of this paper for this....

1. (120) Show that $\texttt{let x} = 5 \texttt{ in let y} = 10 \texttt{ in } x + y \Downarrow 15$

2. (121) Show that `let x` $= 5$ `in let y` $= 10$ `in` $x * x \Downarrow 25$
3. (122) Show that `let f = fun x->x * x in let y` $= 4$ `in f y` $\Downarrow 16$