Illinois Institute of Technology
Department of Computer Science

CS 330 Discrete Structures

Fall Semester, 2010

MidTerm

Oct. 6th, 2010

1. The quantifier $\exists_n$ denotes "there exists exactly n", so that $\exists_n x P(x)$ means there exist exactly n values in the domain such that $P(x)$ is true. Determine the true value of these statement where the domain consists of all real numbers.
   (a) $\exists_0 x(x^2 = -1)$
   (b) $\exists_1 x(|x| = 0)$
   (c) $\exists_2 x(x^2 = 2)$
   (d) $\exists_3 x(x = |x|)$
2. Prove that if A and B are countable sets, then $A \times B$ is also a countable set.
3. (a) How can you produce a big-O estimate for a function that is the sum of different terms where each term is the product of several functions?
   (b) Give a big-O estimate for the function $f(n) = (n! + 1)(2^n + 1) + (n^{n-2} + 8n^{n-3})(n^3 + 2^n)$. For the function g in your estimate $f(x)$ is $O(g(x))$ use a simple function of smallest possible order.
4. Use mathematical induction to show that given a set of $n+1$ positive integers, none exceeding $2n$, there is at least one integer in this set that divides another integer in the set.
5. Use strong induction to show that every positive integer n can be written as a sum of distinct powers of two, that is, as a sum of a subset of the integers $2^0 = 1, 2^1 = 2, 2^2 = 4$, and so on. [*Hint:* For the inductive step, separately consider the case where $k + 1$ is even and where it is odd. When it is even, note that $(k + 1)/2$ is an integer.]