Illinois Institute of Technology
Department of Computer Science

CS 330 Discrete Structures

Fall Semester, 2010

Final

In Class

1. The sequence of **Lucas number** is defined by $l_0 = 2, l_1 = 1$, and $l_n = l_{n-1} + l_{n-2}$ for $n = 2, 3, 4, \cdots$ Show that $f_n + f_{n+2} = l_{n+1}$ whenever n is a positive integer, where f_i and l_i are the ith Fibonacci number and ith Lucas number, respectively.

2. (a) How many different strings can be made from the word *PEPPERCORN* when all the letters are used?
   (b) How many of these strings start and end with the letter P?
   (c) In how many of these strings are the three letter Ps consecutive?

3. A witness to a hit-and-run accident tells the police that the license plate of the car in the accident, which contains three letters followed by three digits, starts with the letters AS and contains both the digits 1 and 2. How many different license plates can fit this description?

4. Suppose that n balls are tossed into b bins so that each ball is equally likely to fall into any of the bins and that the tosses are independent.
   (a) Find the probability that a particular ball lands in a specified bin.
   (b) What is the expected number of balls that land in a particular bin?
   (c) What is the expected number of balls tossed until a particular bin contains a ball?
   (d) What is the expected number of balls tossed until all bins contain a ball? [Hint: Let X_i denote the number of tosses required to have a ball land in an ith bin once $i-1$ bins contain a ball. Find $E(X_i)$ and use the linearity of expectations.]

5. A small post office has only 4-cent stamps, 6-cent stamps, and 10-cent stamps. Find a recurrence relation for the number of ways to form postage of n cents with these stamps if the order that the stamps are used matters. What are the initial conditions for this recurrence relation?

6. A relation R is called **circular** if aRb and bRc imply that cRa. Show that R is reflexive and circular if and only if it is an equivalence relation.

7. Must an asymmetric relation also be antisymmetric? Must an antisymmetric relation must be asymmetric? Give reasons for your answers. (I do not like this question, because we are open book exam... – Shiguang)

8. Suppose that G is a connected multigraph with $2k$ vertices of odd degree. Show that there exist k subgraphs that have G as their union, where each of these subgraphs has an Euler path and where no two of these subgraphs have an edge in common. [Hint: Add k edges to the graph connecting pairs of vertices of odd degree and use an Euler circuit in this larger graph.]

9. For which values of m and n does the complete bipartite graph $K_{m,n}$ have an

(a) Euler circuit?

(b) Euler path?