Illinois Institute of Technology
Department of Computer Science

CS 330 Discrete Structures

Fall Semester, 2010

Quiz 1

Due date: 10th September, 2010

1. Construct a truth table for each of these compound propositions.

   - $(p \wedge q) \rightarrow (p \vee q)$
   - $(q \rightarrow \neg p) \leftrightarrow (p \leftrightarrow q)$

   solution:

p	q	$p \vee q$	$p \wedge q$	$(p \wedge q) \rightarrow (p \vee q)$
T	T	T	T	T
T	F	T	F	T
F	T	T	F	T
F	F	F	F	T

p	q	$\neg p$	$q \rightarrow \neg p$	$p \leftrightarrow q$	$(q \rightarrow \neg p) \leftrightarrow (p \leftrightarrow q)$
T	T	F	F	T	F
T	F	F	T	F	F
F	T	T	T	F	F
F	F	T	T	T	T

2. The police have three suspects for the murder of Mr. Cooper: Mr. Smith, Mr. Jones, and Mr. Williams. Smith, Jones, and Williams each declare that they did not kill Cooper. Smith also states that Cooper was a friend of Jones and that Williams disliked him. Jones also states that he did not know Cooper and that he was out of town the day Cooper was killed. Williams also states that he saw both Smith and Jones with Cooper the day of the killing and that either Smith or Jones must have killed him. Can you determine who the murderer was if

   (a) one of the three men is guilty, the two innocent men are telling the truth, but the statements of the guilty man may or may not be true?

   (b) innocent men do not lie?

   solution:

   (a) We look at the three possibilityes of who the innocent men might be. If Smith and Jones are innocent (and therefore telling the truth), then we get an immediate contradiction, since Smith said that Jones was a friend of Cooper, but Jones said that he did not even know Cooper. If Jones and Williams are the innocent truth-tellers, then we again get a contradiction, since Jones says that he did not know Cooper and was out of town, but Williams says he saw Jones with Cooper (presumably in town, and presumably if he was with him, then he knew him). Therefore it must be the case that Smith and Williams are telling the truth. Their statements do not contradict each other. Based on William' statement, we know taht Jones is lying, since he said taht he did not know Cooper when in fact he was with him. Therefore Jones is the murderer.

(b) This is just like part (a), except that we are not told ahead of time that one of the men is guilty. Can non of them be guilty? If so, then they are all telling the truth, but this is imipossible, because as we just saw, some of the statements are contradictory. Can more than one of them be guilty? If, for example, they are all guilty, then their statements give us no information. So that is certainly possible.