Illinois Institute of Technology
Department of Computer Science

CS 330 Discrete Structures

Fall Semester, 2010

Homework Assignment 2

Due date: Sep. 10th, 2010 (23:59)

1. (#48 in page 49)

   Establish these logical equivalences, where x does not occur as a free variable in A. Assume that the domain is nonempty.

   (a) $\forall x(A \to P(x)) \equiv A \to \forall x P(x)$

   (b) $\exists x(A \to P(x)) \equiv A \to \exists x P(x)$

   Solution:

   (a) There are two cases. If A is false, then both sides of the equivalence are true, because a conditional statement with a false hypothesis is true. If A is true, then $A \to P(x)$ is equivalent to $P(x)$ for each x, so the left-hand side is equivalent to $\forall x P(x)$, which is equivalent to the right-hand side.

   (b) There are two cases. If A is false, then both sides of the equivalence are true, because a conditional statement with a false hypothesis is true (and we are assuming that the domain is nonempty). If A is true, then $A \to P(x)$ is equivalent to $P(x)$ for each x, so the left-hand side is equivalent to $\exists x P(x)$, which is equivalent to the right-hand side.

2. (#62 in page 50) Let $P(x), Q(x), R(x)$, and $S(x)$ be the statements "x is a duck," "x is one of my poultry," "x is an officer," and "x is willing to waltz," respectively. Express each of these statements using quantifiers; logical connectives; and $P(x), Q(x), R(x)$, and $S(x)$.

   (a) No ducks are willing to waltz.

   (b) No officers ever decline to waltz.

   (c) All my poultry are ducks.

   (d) My poultry are not officer.

   (e) Does (d) follow from (a), (b), and (c)? If not, is there a correct conclusion?

   solution

   (a) $\forall x(P(x) \to \neg S(x))$

   (b) $\forall x(R(x) \to S(x))$

   (c) $\forall x(Q(x) \to P(x))$

   (d) $\forall x(Q(x) \to \neg R(x))$

   (e) Yes. If x is one of my poultry, then x is a duck (by (c)), hence not willing to waltz (by (a)). Since officers are always willing to waltz (by (b)), x is not an officer.

3. (#18 in page 60) Express each of these system specifications using predicates, quantifiers, and logical connectives, if necessary.

   (a) At least one console must be accessible during every fault condition.

   (b) The e-mail address of every user can be retrieved whenever the archive contains at least one message sent by every user on the system.

   (c) For every security breach there is at least one mechanism that can detect that breach if and only if there is a process that has not been compromised.

   (d) There are at least two paths connecting every two distinct endpoints on the network.

   (e) No one knows the password of every user on the system except for the system administrator, who knows all passwords.

   solution:

   (a) $\forall f(H(f) \rightarrow \exists c A(c))$, where $A(x)$ means that console x is accessible, and $H(x)$ means that fault condition x is happening.

   (b) $(\forall u \exists m(A(m) \wedge S(u,m))) \rightarrow \forall u R(u)$, where $A(x)$ means that the archive contains message x, $S(x,y)$ means that user x sent message y, and $R(x)$ means that the e-mail address of user x can be retrieved.

   (c) $(\forall b \exists m D(m,b)) \leftrightarrow \exists p \neg C(p)$, where $D(x,y)$ means that mechanism x can detect breach y, and $C(x)$ means that process x has been compromised.

   (d) $\forall x \forall y(x \neq y \rightarrow \exists p \exists q(p \neq q \wedge C(p,x,y) \wedge C(q,x,y)))$, where $C(p,x,y)$ means that path p connects endpoint x to endpoint y.

   (e) $\forall x((\forall u K(x,u)) \leftrightarrow x = SysAdm)$,where $K(x,y)$ means that person x knows the password of user y.

4. (#24 in page 74) Identify the error or errors in this argument that supposedly shows that if $\forall x(P(x) \vee Q(x))$ is true then $\forall x P(x) \vee \forall x Q(x)$ is true.

1. $\forall x(P(x) \vee Q(x))$	Premise
2. $P(c) \vee Q(c)$	Universal instantiation from (1)
3. $P(c)$	Simplification from (2)
4. $\forall x P(x)$	Universal generalization from (3)
5. $Q(c)$	Simplification from (2)
6. $\forall x Q(x)$	Universal generalization from (5)
7. $\forall x(P(x) \vee \forall x Q(x))$	Conjunction from (4) and (6)

   solution: Step 3 and 5 are incorrect; simplification applies to conjunctions, not disjunctions.

5. (#28 in page 74)

   Use rules of inference to show that if $\forall x(P(x) \vee Q(x))$ and $\forall x((\neg P(x) \wedge Q(x)) \rightarrow R(x))$ are true, then $\forall x(\neg R(x) \rightarrow P(x))$ is also true, where the domains of all quantifiers are the same.

   solution: We want to show that the conditional statement $\neg R(a) \rightarrow P(a)$ is true for all a in the domain; the desired conclusion then follows by universal generalization. Thus we want to show that if $\neg R(a)$ is true for a particlualr a, then $P(a)$ is also true. For such an a, universal modus tollens applied to the second premise gives us $\neg(\neg P(a) \wedge Q(a))$. By rules from propositional logic, this gives us $P(a) \vee \neg Q(a)$. By universal generalization from the first premise, we have $P(a) \vee Q(a)$. Now by resolution we can conclude $P(a) \vee P(a)$, which is logically equivalent to $P(a)$, as desired.