Illinois Institute of Technology
Department of Computer Science

# CS 330 Discrete Structures

Fall Semester, 2010

## Homework Assignment 5

Due date: 25th Oct., 2010

1. (#28 in page 345) How many license plates can be made using either three letters followed by three digits or four letters followed by two digits?

   **Solution:** $26^3 10^3 + 26^4 10^2$.

2. (# 36 in page 345) How many partial functions (see the preamble to Exercise 73 in Section 2.3) are there from a set with five elements to sets with each of these number of element?

   (a) 1
   (b) 2
   (c) 5
   (d) 9

   **Solution:** Because the functions could be partial, so if the range has $n$ elements, there will be $n+1$ choices of mappings (choose one of them or do not choose anything.). So the ways of mapping will be $(n+1)^N$, where $N$ is the size of domain. So $N = 5$ in our problem.

   So the answer is $2^5, 3^5, 6^5, 10^5$, respectively.

3. (#40 in page 346) In how many ways can a photographer at a wedding arrange 6 people in a row from a group of 10 people, where the bride and the groom are among these 10 people, if

   (a) the bride must be in the picture?
   **Solution:** We first place the bride in any of the 6 positions. Then, from left to right in the remaining positions, we choose the other five people to be in the picture; this can be done in $9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 = 15120$ ways. Therefore the answer is $6 \cdot 15120 = 90,720$.

   (b) both the bride and groom must be in the picture?
   **Solution:** We first place the bride in any of the 6 positions, and then place the groom in any of the 5 remaining positions. Then, from left to right in the remaining positions, we choose the other four people to be in the picture; this can be done in $8 \cdot 7 \cdot 6 \cdot 5 = 1680$ ways. Therefore the answer is $6 \cdot 5 \cdot 1680 = 50,400$.

   (c) exactly one of the bride and the groom is in the picture?
   **Solution:** From part (a) there are 90720 ways for the bride to be in the picture. There are $90720 - 50400 = 40320$ ways for just the bride to be in the picture. Symmetrically, there are 40320 ways for just the groom to be in the picture. Therefore the answer is $40320 + 40320 = 80,640$.

4. (# 52 in page 346) Use a tree diagram to find the number of bit strings of length four with no three consecutive 0s. **Solution:** We draw the tree, with its root at the top. We show a branch for each of the possibilities 0 and 1, for each bit in order, except that we do not allow three consecutive 0's. Since there are 13 leaves, the answer is 13.

5. (# 12 in page 353) How many ordered pairs of integers $(a, b)$ are needed to guarantee that there are two ordered pairs $(a_1, b_1)$ and $(a_2, b_2)$ such that $a_1 \text{mod} 5 = a_2 \text{mod} 5$ and $b_1 \text{mod} 5 = b_2 \text{mod} 5$?

   **Solution:** Working modulo 5 there are 25 pairs: $(0, 0), (0, 1), \cdots, (4, 4)$.Thus we could have 25 ordered pairs of integers $(a, b)$ such that no two of them were equal when reduced modulo 5. The pigeonhole principle, however, guaratees that if we have 26 pairs, then at least two of them will have the same coordinates, modulo 5.

6. (# 16 in page 353) How many numbers must be selected from the set $\{1, 3, 5, 7, 9, 11, 13, 15\}$ to guarantee that at least one pair of these numbers add up to 16?

   **Solution:** We can apply the pigeonhole principle by grouping the numbers cleverly into pairs (subsets) that add up to 16, namely $\{1, 15\}, \{3, 13\}, \{5, 11\}, \{7, 19\}$. If we select five numbers from the set $\{1, 3, 5, 7, 9, 11, 13, 15\}$, then at least two of them must fall within the same subset, since there are only four subsets. Two numbersin the same subset are desired pair that add up to 16. We also need to point out that choosing four numbers is not enough, since we could choose $\{1, 3, 5, 7\}$, and no pair of them add up to more than 16.

7. (# 24 in page 354) Show that in a group of five people (where any two people are either friends or enemies), there are not necessarily three mutual friends or three mutual enemies.

   **Solution:**

   Let the people be $A, B, C, D, E$. Suppose the following pairs are friends: $A-B, B-C, C-D, D-E, E-A$. The other five pairs are enemies. In this example, there are no three mutual friends and no three mutual enemies.