Analytical Bounds on Broadcast with Hitch-hiking in Wireless Ad-Hoc Networks

Gruia Calinescu *

Abstract

Recently, there have been papers indicating that the maximal ratio combiner device can result in energy savings in wireless ad hoc networks by using Hitch-hiking. We study the Min-Energy Broadcast with Hitch-hiking problem, an idealized version of broadcast using hitch-hiking, a problem studied experimentally in the INFOCOM 2004 paper of Agarwal et. al. Min-Energy Broadcast with Hitch-hiking captures the maximum savings one can achieve in broadcasting using maximal ratio combiners. We show that the optimum of the classical Min-Energy Broadcast problem is at most $O(\log^2 n)$ times the optimum of Min-Energy Broadcast with Hitch-hiking, where n is the number of nodes in the networks. We show that this bound is tight up to a constant. In the special case when the nodes are on a line and the power requirement for node u to reach node v is $d(u,v)^{\kappa}$, where $d(u,v)$ the Euclidean distance between u and v and κ is the signal attenuation exponent, which is assumed to be in between 2 and 5, we show that the optimum of the Min-Energy Broadcast problem is at most a constant times optimum of Min-Energy Broadcast with Hitch-hiking.

We also show that Min-Energy Broadcast with Hitch-hiking is NP-Hard, and present approximation algorithms. A formal definition of Min-Energy Broadcast with Hitch-hiking is given below. The input consists of a complete directed graph $G = (V, E)$ with power requirement function $c : E \to R^+$, and a source $s \in V$. The output consists of a permutation $\tau =< v_1, v_2, \ldots, v_n >$ of V with $v_1 = s$ and power assignment $p(v)$ of every vertex v. For every $1 \le i < j \le n$, define $q(v_i v_j) = p(v_i)/c(v_i v_j)$. An output is feasible if for every $j > 1$ we have $\sum_{i=1}^{j-1} q(v_i v_j) \ge 1$. The objective is to minimize $\sum_{i=1}^{n} p(v_i)$.

1 Introduction

Wireless ad hoc networks have received significant attention in recent years due to their potential applications in battlefield, emergency disaster relief, and other application scenarios (see, e.g., [13, 16, 19, 18]). Unlike wired networks or cellular networks, no wired backbone infrastructure is installed in wireless ad hoc networks. A communication session is achieved either through single-hop transmission if the recipient is within the transmission range of the source node, or by relaying through intermediate nodes otherwise. We assume that omnidirectional antennas are used by all nodes to transmit and receive signals. Thus, a transmission made by a node can be received by all nodes within its transmission range. This feature is extremely useful for energy-efficient multicast and broadcast communications.

One of the major concerns in ad hoc wireless networks is reducing node power consumption. In fact, nodes are usually powered by batteries of limited capacity. Once the nodes are deployed, it is very difficult or even impossible to recharge or replace their batteries in many application scenarios. Hence, reducing power consumption is often the only way to extend network lifetime. For the purpose of energy conservation, each node can (possibly dynamically) adjust its transmitting power, based on the distance to the receiving node and the background noise. Moreover, the energy density of battery has only doubled every 5 to 20 years as shown in [11]. For the purpose of energy conservation, one can use energy-efficient range assignment [4, 18]

*Department of Computer Science, Illinois Institute of Technology, Chicago, IL 60616. E-mail: calinesc@iit.edu.

which aims on assigning minimum transmission energy to the nodes while maintaining required property, e.g. connectivity, and/or power saving protocol which aims on putting nodes into sleep mode [19, 15]. It has been shown that both techniques could save energy resulting in prolonging network lifetime which is the major consequence of energy conservation in wireless ad hoc networks.

In broadcast and multicast scenario, the goal is to have every target node in the networks receive the complete message sending from the source. Broadcast in wireless ad hoc networks is very important since one can use broadcast to request the network change, to discover nodes' neighbors, etc. The energy-efficient broadcast in wireless ad hoc networks will reduce the utiliziition of network resources and thus extend the network lifetime.

Wieselthier et al. [19, 20] introduce the Min-Energy Broadcast problem and propose three heuristics: based on Shortest-Paths Trees, Minimum Spanning Trees (MST), and the Broadcast Incremental Power (BIP) heuristic. BIP constructs the tree by starting with the source node as root and adds at a time one node which is uncovered and can be added to the tree at minimum additional cost. BIP can do this because of the Wireless Multicast Advantage (WMA) meaning that a single transmission from a node can be received by all nodes within the transmission range of sending node.

Since Min-Energy Broadcast is NP-Hard [8, 2], polynomial-time approximation algorithms have been proposed. The approximation ratio of such an algorithm is the maximum of the ratio of the output to the optimum solution; the smaller the ratio the better.

In [18], Wan *et al.* gave analytical results for different algorithms to construct a minimum energy broadcast tree. They proved that the approximation ratio of the MST-nased algorithm is between 6 and 12.15 and that of BIP is between $\frac{13}{3}$ and 12.15. Recently, the approximation ratio of the Minimum Spanning Tree algorithm was improved to 7.45 by Flammini et. al [10].

Cagalj *et al.* [2] propose a heuristic called EWMA. It builds an energy efficient broadcast tree by improving an initial feasible solution. This method allows every forwarding node increase its transmission power if the total power of the tree decreases.

None of the previous papers use the *maximal ratio combiner* device in order to save energy. They all assume that a node can decode a signal only when the signal strength is above the threshold and ignore one below.

M. Agarwal *et al.* have proposed in [1] the wireless multicast with *Hitch-hiking* which uses the concept of combining partial signals called *maximal ratio combiner* [14]. If the signal is good or high enough, the receiving node equipped with the mentioned technique would be able to combine the signal with previously received signals to decode the whole message, given that all those signals representing the same message. By this ability, sending nodes may consider decreasing transmitting power if doing so reduces the overall consumed power of the networks, provided that every node still completely receives the original broadcast message. By using this advantage, [1] propose a centralized heuristic called (WMH) for Wireless Multicast with *Hitch-hiking*. They evaluate its performance via simulation showing that *Hitch-hiking* can reduce the transmission power of broadcast by as much as 50% comparing to broadcast without using this technique.

Our work involves mainly quantifying the savings Hitch-hiking can produce.This is useful to the engineer as one needs to balance the cost of combiners (including the more complicated protocols one would need to use Hitch-hiking) with the energy savings obtained by using combiners.

We study the Min-Energy Broadcast with Hitch-hiking (MBH) problem, an idealized version of broadcast using hitch-hiking, and a problem considered from an experimental point of view in Agarwal et. al. [1]. Min-Energy Broadcast with Hitch-hiking captures the maximum savings one can achieve in broadcasting using maximal ratio combiners. Min-Energy Broadcast with Hitch-hiking ignores several engineering constraints, such as interference, timing, timing threshold, and the difficulty in assembling packages. Thus the energy used by an optimum solution of Min-Energy Broadcast with Hitch-hiking would be lower than a real-word optimum. The problem is still important, since it allows us to analytically quantify the savings Hitch-hiking would produce. Future papers might also show how to convert a solution for Min-Energy Broadcast with Hitch-hiking into a real-world broadcast protocol.

We show that the optimum of the classical Min-Energy Broadcast problem is at most $O(\log^2 n)$ times the optimum of Min-Energy Broadcast with Hitch-hiking, where n is the number of nodes in the networks. We show that this bound is tight up to a constant. Following the proofs shows that the upper bound becomes $O(\log n \log \delta)$, where δ is the ratio of the threshold to successfully decode the packet to the threshold to acquire the timing. The $O(\log^2 n)$ bound above holds when the matrix $c(u, v)$ giving the power requirement of node u fully transmitting to node v, is arbitrary (asymmetric power requirements being possible). The $\Omega(\log^2 n)$ holds for symmetric matrices we construct. The proofs use techniques from LP-rounding of Set Cover (as shown, for example, in [17]).

In many situations, the power requirements matrix is not arbitrary. In the Euclidean case, $c(u, v) = ||u, v||^\kappa$, where $||u, v||$ is the Euclidean distance and κ is a constant between 2 and 5. The Euclidean case frequently makes power assignment problems easier - for example Min-Energy Broadcast has a constant approximation ratio [18, 7, 10]. An even more restricted case is the line case - when the power requirements are given as above and the nodes lie on a line. In the line case we show that the optimum of the Min-Energy Broadcast problem is at most a constant times optimum of Min-Energy Broadcast with Hitch-hiking. We further conjecture that a constant ratio also holds in the Euclidean case.

We also study Min-Energy Broadcast with Hitch-hiking as an optimization problem. We show that it is NP-Hard. The bounds mentioned above together with previously published algorithms give us an $O(\log^3 n)$ approximation ratio for the problem and $O(\log^2 n)$ approximation ratio in the Euclidean case. In the line case, we present a very simple 2-approximation algorithm.

The rest of the paper is organized as follows. In Section 2 we formally introduce the Min-Energy Broadcast with Hitch-hiking problem and describe its relationship with previous work. In Section 3 we present our $\Omega(\log^2 n)$ bound on the rate of savings obtained by hitchhiking. Section 4 presents a 2-approximation algorithm for Min-Energy Broadcast with Hitch-hiking in the line case and the constant bound on the ratio between the optimae with and without hitchhiking. Section 5 considers Min-Energy Broadcast with Hitch-hiking as an optimization problem, providing the NP-Hardness proof and approximation algorithm. In Section 6 we discuss generalizations and open problems.

2 Preliminaries

We introduce the Min-Energy Broadcast with Hitch-hiking problem as an idealized version of the problem considered by Agarwal et. al [1]. In a wireless ad-hoc network, the signal transmitted by a node can reach any receiver as long as the channel gain between the transmitter and the receiver is not zero. When node u transmits with power $p(u)$, node v can receive the signal with the signal-to-noise ratio

$$\gamma_{uv} = \frac{p(u)}{c(u, v)} \tag{1}$$

where $c(u, v)$ is the noise density divided by the gain of the wireless link from u to v. $c(u, v)$ is a quantity which depends on the nodes u and v and their position, and we assume it is known as an input to the problem. The formula above is a simplification not taking in account the interference from the other nodes transmitting at the same time on the same frequency. Agarwal et. al [1] argue that this simplification is valid when the processing gain is very large or a scheduling algorithm is designed to avoid collisions.

The Hitch-hiking technique is based on partial reception. Agarwal et. al. [1] introduce two thresholds: γ_p for successfully decoding the payload of a packet, and γ_{ack} to successfully acquire timing. Following

[1], when node u transmits a packet with power $p(u)$, the amount of reception by node v is given by:

$$q_{uv} = \begin{cases} 0 & \text{if } \gamma_{uv} \leq \gamma_{ack} \\ \frac{\gamma_{uv}}{\gamma_p} & \text{if } \gamma_{ack} < \gamma_{uv} \leq \gamma_p \\ 1 & \text{if } \gamma_{uv} \geq \gamma_p \end{cases}$$

After packets are partially received, they are combined by a maximal ratio combiner [14]. For example, if node v receives the same packet from u_1 and u_2 with an amount of reception $q_{u_1 v}$ and $q_{u_2 v}$, v receives $q_{u_1 v} + q_{u_2 v}$ amount of reception. Following Agarwal et. al [1], we allow such a combination to successfully decode the packet if $\sum_{u:u \text{ transmits the packet}} q_{uv} \geq 1$. This technique of partial reception and combination is called Hitch-hiking.

With hitchhiking the broadcast does not proceed on a tree rooted at the source; instead a directed acyclic graph (given by edge set D - all the nodes are included) is used, with an edge uv meaning that u sends the packet and v decodes the packet partially based on the signal of u. Successful decoding by node v is then given by $\sum_{uv \in D} q_{uv} \geq 1$.

We present an example (also illustrated in Figure 1) showing an instance and a feasible solution. Say s sends a packet to v_1 and v_2, using power $p(s) = c(sv_2)$. In our example, v_3 is such that $c(sv_3) = 3 \cdot p(s)$. However, v_3 can still decode a fraction of the packet sent by s. Precisely, if v_1 also sends with power $p(v_1) = \frac{2}{3}c(v_1 v_3)$ the original packet, then using the combiner device, v_3 can put together the packets of s and v_1 to receive the whole message.

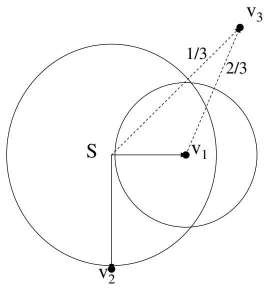

Figure 1: An illustration of the advantage of hitch-hiking. Node v_3 receives $1/3$ of the information from s and $2/3$ from v_1

We make the problem simpler by setting $\gamma_{ack} = 0$ and define Min-Energy Broadcast with Hitch-hiking (MBH) below. Such a simplification is also made by Agarwal et. al [1] before formally defining the problem. They used a set of "forwarding nodes", we prefer the equivalent definition below.

Min-Energy Broadcast Hitch-hiking: The input consists of a complete directed graph $G = (V, E)$ with power requirement function $c : E \to R^+$, and a source $s \in V$. The output consists of a permutation $\tau =< v_1, v_2, \ldots, v_n >$ of V with $v_1 = s$ and power assignment $p(v)$ of every vertex v. For every $1 \leq i < j \leq n$, let $q(v_i v_j) \leq p(v_i)/c(v_i v_j)$. An output is feasible if for every $j > 1$ we have $\sum_{i=1}^{j-1} q(v_i v_j) \geq 1$. The objective is to minimize $\sum_{i=1}^{n} p(v_i)$. So in this formulation we put a fraction $q(v_i v_j) \geq 0$ on every pair

$i < j$, not only on the edges of the directed acyclic digraph with edge set D. One can put 0 fractions on pairs not in D by first doing a topological sort of the directed acyclic digraph. This formulation also allows $q(v_i v_j)$ to exceed 1; this does not change the problem as in any such solution one can reduce $q(v_i v_j)$ to 1 and still have a feasible solution.

Note that once the permutation τ is given, the best values for $p(v_i)$ are given by solving a linear program with variable $p(v_i)$ for all i and $q(v_i v_j)$ for all $i < j$. We call this linear program LP_1:

$$\text{Minimize} \sum_{v \in V} p(v) \text{ subject to}$$

$$q(v_i v_j) c(v_i v_j) \leq p(v_i) \quad \forall 1 \leq i < j \leq n \tag{2}$$

$$\sum_{i=1}^{j-1} q(v_i v_j) \geq 1 \quad \forall j \geq 2. \tag{3}$$

$$q(v_i v_j) \geq 0 \quad \forall 1 \leq i < j \leq n \tag{4}$$

$$p(v_i) \geq 0 \quad \forall 1 \leq i \leq n \tag{5}$$

The difficulty of MBH consists in selecting the permutation.

3 The ratio of optimae of Min-Energy Broadcast with Hitch-hiking and Min-Energy Broadcast

In this section we show that the optimum of the classical Min-Energy Broadcast problem does not exceed $O(\log^2 n)$ times the optimum of the MBH problem on the same instance. We also construct instances where the optimum without Hitch-hiking is $\Omega(\log^2 n)$ higher than the optimum with Hitch-hiking. The proofs contain several arguments from the $\Theta(\lg n)$ integrality gap of the linear programming relaxation of Set Cover as described in the book of Vazirani [17]. The theorem below provides the upper bound by using an optimum solution for the Min-Energy Broadcast with Hitch-hiking problem.

Theorem 1 *Given an instance of Min-Energy Broadcast with Hitch-hiking and a feasible solution of total power p, there is feasible solution to the same instance of Min-Energy Broadcast with total power at most $O(\log^2 n)p$, where n is the number of nodes in the instance.*

Proof: Renumber the vertices of V in the order given by τ, the permutation of the feasible solution with Hitch-hiking, and p_i be the power of the i^{th} vertex.

The proof is based on a probabilistic argument. A new power function $\overline{p}(v)$ for $v \in V$ is constructed. For $1 \leq i < j \leq n$, edges $v_i v_j$ are said to be *selected* when $\overline{p}(v_i) \geq c(v_i v_j)$, or in other words the packet send by v_i would be decoded completely by v_j. For $j > 1$, we call the vertex v_j *covered* if there some $i < j$ with $v_i v_j$ selected. One can easily check that if every vertex is covered, then we have a feasible solution to the Min-Energy Broadcast problem.

We assign power $\overline{p}_i = \overline{p}(v_i)$ as follows: for each i, independently pick $x_i \in (0, 1)$. If $x_i < \frac{1}{2n}$, set $\overline{p}_i = 2p_i n$. If $x_i \geq \frac{1}{2n}$, set $\overline{p}_i = \frac{2p_i}{x_i}$.

Then the expected value

$$E[\overline{p}_i] = \frac{1}{2n} \cdot 2p_i n + \int_{1/2n}^{1} \frac{2p_i}{x_i} = p_i + 2p_i(\ln 1 - \ln \frac{1}{2n}) = p_i(1 + 2\ln 2n), \tag{6}$$

and thus the total expected power of the random power assignment $\overline{p}$ is

$$E[\sum_{i=1}^{n} \overline{p}_i] = \sum_{i=1}^{n} E[\overline{p}_i] = (1 + 2\ln 2n) \sum_{i=1}^{n} p_i. \tag{7}$$

If vertices u and v satisfy $\frac{1}{2n} \leq q(uv) \leq \frac{1}{2}$, we have that the probability edge uv is selected is equal to

$$Pr[\overline{p}(u) \geq c_{uv}] \geq Pr[\overline{p}(u) \geq \frac{p(u)}{q(uv)}] = Pr[\frac{2p(u)}{x_u} \geq \frac{p(u)}{q(uv)}] = Pr[x_u \leq 2q(uv)] = 2q(uv), \quad (8)$$

where we used the constraint $q(uv) \leq \frac{p(u)}{c_{uv}}$, the fact that $\frac{p(u)}{q(uv)} \leq 2np(u)$ and the fact that x_u is uniformly picked from the interval $(0, 1)$. Thus for every u with u before v in τ, the probability uv is selected is at least:

$$z_{uv} = \begin{cases} 1 \text{ if } q(uv) > \frac{1}{2} \\ 2q(uv) \text{ if } \frac{1}{2n} \leq q(uv) \leq \frac{1}{2} \\ 0 \text{ if } q(uv) < \frac{1}{2n} \end{cases}$$

From now on we follow the exposition from Vazirani's book [17] closely in computing the probability that a node v is covered by a selected edge. Since $\sum_u q(uv) \geq 1$, and we have n nodes, $\sum_{u:q(uv)\geq\frac{1}{2n}} q(uv) \geq \frac{1}{2}$ and therefore $\sum_u z_{uv} \geq 1$. Using elementary calculus, it is easy to show that under this condition, the probability that v is covered by selected edges is minimized when each of the z_{uv} is $1/n$. Using the fact that each vertex u independently chooses $\overline{p}(u)$, we have:

$$Pr[v \text{ is covered by selected edges}] \geq 1 - \left(1 - \frac{1}{n}\right)^n \geq 1 - \frac{1}{e},$$

where e is the base of natural logarithms. Hence each node is covered with constant probability by the selected edges.

To get a complete cover, independently pick $c \log n$ such x_u for every u, and among them use the largest value when computing $\overline{p}(u)$, where c is a constant such that

$$\left(\frac{1}{e}\right)^{c \log n} \leq \frac{1}{4n}.$$

Now,

$$Pr[v \text{ is not covered by selected edges}] \leq \left(\frac{1}{e}\right)^{c \log n} \leq \frac{1}{4n}.$$

Summing over all nodes $v \in V$, we get

$$Pr[\text{there is vertex } v \text{ not covered by selected edges}] \leq n \cdot \frac{1}{4n} \leq \frac{1}{4}.$$

From Equation 7, we obtain $E[\sum_{i=1}^n \overline{p}_i] \leq c \log n(1 + 2 \ln 2n) \sum_{i=1}^n p_i$. Applying Markov's Inequality we obtain

$$Pr[\sum_{i=1}^n \overline{p}_i \geq 4c \log n(1 + 2 \ln 2n) \sum_{i=1}^n p_i] \leq \frac{1}{4}. \quad (9)$$

The probability of the union of the two undesirable events is $\leq 1/2$. Hence,

$$Pr[\sum_{i=1}^n \overline{p}_i \leq 4c \log n(1 + 2 \ln 2n) \sum_{i=1}^n p_i \text{ and every } v \text{ is covered by selected edges}] \geq \frac{1}{2}. \quad (10)$$

Thus there exist a valid power assignment without hitch-hiking of total power at most $4c \log n(1 + 2 \ln 2n) \sum_{i=1}^n p_i$, completing the proof of the theorem. ■

The proof above can be made into a randomized polynomial-time algorithm; however we do not see the benefit of such an algorithm. Also note that if every nonzero $q(uv)$ is at least $\frac{1}{\delta}$, then the same proof gives a $O(\log n \log \delta)$ ratio in between the two optimum values. Depending on the values of n and δ, the $O(\log n \log \delta)$ ratio might be better than the trivial bound of δ. Indeed, blowing the power of every node by a factor of 1δ always yields a feasible solution without hitch-hiking from a solution with hitch-hiking with all $q(uv) \geq \frac{1}{\delta}$.

Interesting enough, the bound in Theorem 1 is tight up to a constant.

Theorem 2 *There exist instances with n nodes where the ratio of the optimum to Min-Energy Broadcast to the optimum of Min-Energy Broadcast with Hitch-hiking is $\Omega(\log^2 n)$.*

Proof: We repeatedly use the construction from Example 13.4 of [17]. For a positive integer k, the construction gives a bipartite graph H with the vertex set $V(H) = A \cup B$, where A and B are disjoint and $|A| = |B| = 2^k - 1$, and every vertex of B has degree 2^{k-1}. Moreover, any subset $S \subseteq A$ which satisfies the condition that every vertex in B has a neighbor in S must have at least k vertices.

We construct the instance with symmetric power requirements ($c(uv) = c(vu)$ for all u, v) as follows. We fix an even integer parameter $q \geq 8$. The nodes are given by $\{s\} \cup A_0 \cup \cup_{j=q/2}^{q} B_j$, where the sets above are disjoint. A_0 has $2^q - 1$ elements and each B_j has $2^{q-j}(2^j - 1)$ elements. We put edges of power requirement 1 between s and all the vertices of A. For all j, we partition B_j into 2^{q-j} sets B_j^i, each of size $2^j - 1$. Also for each j we pick 2^{q-j} disjoint sets with $(2^j - 1)$ elements A_j^i, all subset of A_0 (note that A_0 has enough elements). For all j and i we use the construction from Example 13.4 to connect the vertices of A_j^i to the vertices of B_j^i, with A_j^i being A and B_j^i being B. An edge between a vertex of A_j^i and a vertex of B_j^i has power requirement 2^j. All the remaining power requirements are ∞.

Note that $n = 1 + 2^q - 1 + \sum_{j=q/2}^{q}(2^{q-j}(2^j - 1)$ and thus $q = \Theta(\log n)$. A solution with hitchhiking is to start the permutation with s followed by the vertices of A_0, and assign power $p(s) = 1$ and $p(v) = 2$ for $v \in A_0$. The remaining vertices follow in arbitrary order and have power 0. If a vertex $v \in A_0$ is adjacent to a vertex $y \in B_j$ by an edge of finite power requirement (so $c(vy) = 2^j$), then we set $q(vy) = \frac{2}{2^j} = \frac{1}{2^{j-1}}$. As every vertex of B_j is adjacent to 2^{j-1} vertices of A_0 by edges having $q = \frac{1}{2^{j-1}}$, we see that we have a valid feasible solution with hitch-hiking of total power $2 \cdot (2^q - 1) + 1$.

Now we consider solutions without hitchhiking. Every B_j^i requires j vertices of A_j^i to have power at least 2^j. Fixing j and summing over i, we obtain that A_0 must have $j \cdot 2^{q-j}$ vertices of power at least 2^j. Thus the power of a solution without hitchhiking must exceed

$$\sum_{j=q/2}^{q-1} 2^j (j 2^{q-j} - (j+1) 2^{q-(j+1)})$$

where we took into account the fact that vertices chosen at power $2^{q-(j+1)}$ already have power at least 2^j. Algebraic manipulation gives that the power of a solution without hitchhiking must exceed

$$\sum_{j=q/2}^{q-1} 2^j 2^{q-(j+1)}(j-1) = 2^{q-1} \sum_{j=q/2}^{q-1} (j-1) \geq 2^{q-1} \sum_{j=q/2}^{q-1} \frac{q}{3} \geq 2^{q-1} \frac{q^2}{9}$$

where we used $q \geq 8$. Thus the ratio between the optimum without hitch-hiking to the optimum with hitchhiking can exceed $\frac{q^2}{36}$. Recall that $q = \Theta(\log n)$ and thus the proof is finished. ■

4 The line case

Here we consider the case when the nodes are on a line, and the power requirements are given by $c(u, v) = d(u, v)^\kappa$, where $d(u, v)$ the Euclidean distance between u and v and κ is the signal attenuation exponent, which is assumed to be in between 2 and 5 and is the same for all pairs of nodes.

Our main result relates the optimum with hitchhiking to the optimum without hitchhiking. We don't know how to find the optimum with hitchhiking in polynomial time, but we suspect it is possible We present a simple 2-approximation algorithm below. The ratio of this algorithm also helps in the main result of the section.

Consider the case when the source is the leftmost node on the line. In this case it is easy to show that the best permutation Π enumerates the nodes from left to right, and so optimum is just given by the linear program LP_1.

Consider an instance I with the source is in the middle of the line and let opt_I be the value of the optimum solution. Construct two instances, I_L obtained by keeping only the source and the nodes to the left of the source, and I_R obtained by keeping the source and nodes to the right of the source. Let the two instances have optimum opt_L and opt_R. Solve both instances exactly as in the paragraph above, and define the power of the source as the maximum of its power in the two optimum solutions. We have $opt_I \geq opt_L$ (and symetrically, $opt_I \geq opt_R$) since any solution to I can be transformed into one of I_L without increasing the total power as follows: the order of broadcast in I_L is from right to left and the power levels are the same for all nodes except the source, whose power is set to be the sum of powers used in the solution of I by the source and by the nodes to the right of the source. The solution returned by the algorithm has total power at most $opt_L + opt_R$, and thus is a 2-approximation.

One can also obtain, by a similar argument, a 2-approximate solution by selecting in I as Π any permutation which obeys the following property: if two nodes appear to the left of the source, then the closest appears first in the permutation, and the same property holds for any two nodes to the right of the source. Then solve LP_1 for the given Π.

We continue with the main result of the section:

Theorem 3 *There is a constant c such that the optimum solution to Min-Power Broadcast is at most c times the optimum solution to Min-Power Hitchhiking Broadcast.*

Proof: In light of the argument in the 2-approximation given above, by losing a factor of 2 it is enough to concentrate on instances with the source v_1 as the leftmost node. Let $x_1, x_2, \ldots, x_n$ be the coordinate of vertices $v_1, \ldots, v_n$ sorted from left to right. For $i = 1, 2, \ldots, n-1$, let $a_i = x_{i+1} - x_i$ and for $1 \leq i \leq j \leq n-1$ let $s_{i,j} = x_{j+1} - x_i = \sum_{q=i}^{j} a_q$.

To simplify the proof we fix $\kappa = 2$; the constant c turns out to be smaller as k increases as c is related to the converging series $\sum_{i=1}^{\infty} \frac{1}{i^\kappa}$. For intuition, it is useful to check the example $a_1 = a_2 = \ldots = a_{n-1} = 1$. In fact, we were not able to construct an example with bigger ratio in between the two optimae.

The optimum solution without hitchhiking is given by the summation:

$$L := \sum_{i=1}^{n-1} a_i^2 \tag{11}$$

.

First recall that the permutation Π used in the solution with hitchhiking enumerates the nodes from left to right. The linear program LP_1 has a particularly simple greedy solution: for $i = 1, 2, \ldots, n-1$, set

$$p_i = a_i^2 \left(1 - \sum_{j=1}^{i-1} \frac{p_j}{s_{j,i}^2} \right), \tag{12}$$

where we use the convention that an empty summation is 0.

In words, p_i is the minimum possible power we can assign to node v_i to "reach" node v_{i+1} (that is, $\sum_{j\leq i} q_{j,i+1} = 1$), and note that $p_i \geq 0$ since v_i is barely reached ($\sum_{j\leq i-1} q_{j,i} = 1$) and therefore v_{i+1} is either unreached or barely reached (this happens when $x_i = x_{i+1}$), in which case $p_i = 0$.

To show that p is optimal, take any other feasible solution (p', q') which assigns $p'_i \neq p_i$ to some node v_i and assume i is the first node where this happens. We must have $p'_i > p_i$ since otherwise $\sum_{j\leq i} q'_{j,i+1} < 1$ (this follows from the definition of p_i). We transform p' as follows: we know that v_{i+1} has $\sum_{j\leq i} q'_{j,i+1} > 1$. We reduce p'_i by ε and increase p'_{i+1} by ε, obtaining again a feasible solution without increasing the total value of p'. Appropriately picking $\varepsilon > 0$ will make $p'_i = p_i$, and the transformation is continued until $p'_i = p_i$ for all $i < n-1$. Now, if necessary, we can simply simply reduce p'_{n-1} to p_{n-1}. Thus we proved that p_i the optimum solution to LP_1. Therefore the value of the optimum with hitchhiking is:

$$R := \sum_{i=1}^{n-1} p_i. \tag{13}$$

Our goal is to find constant c and prove $L \leq cR$. We need the following claim:

Claim 4 *Given nonnegative numbers $b_1, b_2, \ldots, b_n$, with b_1 being the biggest, and let $s_i = \sum_{j=1}^{i} b_i$. Then*

$$\sum_{i=1}^{n} \frac{b_i^2}{s_i^2} \leq \frac{\pi^2}{6}$$

.

We prove the claim. Euler has proved that the series $\sum_{i=1}^{\infty} \frac{1}{i^2}$ converges to $\frac{\pi^2}{6}$ and thus the claim is true when $b_1 = b_2 = \ldots = b_n$. When $b_1 \geq b_2 \geq \ldots \geq b_n$, then $\frac{b_i^2}{s_i^2} \leq \frac{b_i^2}{(i \cdot b_i)^2} = \frac{1}{i^2}$ and thus the claim also holds. If $b'_i s$ are not sorted in descending order, we use bubble sort to sort them. Bubble sort involves only swapping adjacent elements and we show below that such a swap only increases the summation. We are going to show that for $s \geq a \geq b$

$$\frac{a^2}{(s+a)^2} + \frac{b^2}{(s+a+b)^2} \geq \frac{b^2}{(s+b)^2} + \frac{a^2}{(s+b+a)^2}, \tag{14}$$

and then plugging $s = \sum_{j=1}^{i-1} a_j$, $a = a_{i+1}$, and $b = a_i$ allows us to swap whenever $a_{i+1} > a_i$ (note that only for $i > 1$ we can have $a_{i+1} > a_i$, and also $s \geq a$). Indeed, Equation 14 is equivalent to

$$\frac{a^2}{(s+a)^2} - \frac{b^2}{(s+b)^2} \geq \frac{a^2}{(s+b+a)^2} - \frac{b^2}{(s+a+b)^2}, \tag{15}$$

which is equivalent to

$$\left(\frac{a}{s+a} + \frac{b}{s+b}\right)\left(\frac{a}{s+a} - \frac{b}{s+b}\right) \geq frac(a+b)(a-b)(s+a+b)^2 \tag{16}$$

which is equivalent to

$$\frac{(as+ab+bs+ba)(as+ab-bs-ba)}{(s+a)^2(s+b)^2} geqfrac(a+b)(a-b)(s+a+b)^2. \tag{17}$$

Since $a \geq b$, we simplify by $(a-b)$ and expand the terms to obtain that Inequation 14 is equivalent to

$$\frac{s(sa+sb+2ab)}{(s^2+2sa+a^2)(s^2+2sb+b^2)} \geq \frac{a+b}{s^2+2sa+2sb+a^2+2ab+b^2} \tag{18}$$

which is equivalent to

$$s^4a + s^4b + 2s^3ab + 2s^3a^2 + 2s^3ab + 4s^2a^2b + 2s^3ab + 2s^3b^2 + 4s^2ab^2 + s^2a^3 + s^2a^2b + 2sa^3b + 2s^2a^2b + 2s^2ab^2 + 4sa^2b^2 + s^2ab^2 + s^2b^3 + 2sab^3 \geq s^4a + s^4b + 2s^3ab + 2s^3b^2 + s^2ab^2 + s^2b^3 + 2s^3a^2 + 2s^3ab + 4s^2a^2b + 4s^2ab^2 + 2sa^2b^2 + 2sab^3 + s^2a^3 + s^2a^2b + 2sa^3b + 2sa^2b^2 + a^3b^2 + a^2b^3.$$

Simplifying the terms we obtain

$$2s^3ab + 2s^2ab^2 + 2s^2a^2b \geq a^3b^2 + a^2b^3, \tag{19}$$

which holds since $s \geq a \geq b$. Thus Inequation 14 holds and Claim 4 follows.

We continue with the proof of Theorem 3. We have:

$$p_i = a_i^2 \left(1 - \sum_{j:j<i \text{ and } a_j<a_i} \frac{p_j}{s_{j,i}^2} - \sum_{j:j<i \text{ and } a_j \geq a_i} \frac{p_j}{s_{j,i}^2} \right), \tag{20}$$

Note that $p_i \leq a_i^2$. Then using Claim 4 for the last inequality:

$$\sum_{j:j<i \text{ and } a_j<a_i} \frac{p_j}{s_{j,i}^2} \leq \sum_{j:j<i \text{ and } a_j<a_i} \frac{a_j^2}{s_{j,i}^2} \leq \sum_{j:j<i \text{ and } a_j<a_i} \frac{a_j^2}{(\sum_{j:j<i \text{ and } a_j<a_i} a_j)^2} \leq \frac{\pi^2}{6} - 1. \tag{21}$$

The -1 term appears since the summation does not include $\frac{a_i^2}{a_i^2}$. Thus

$$p_i \geq a_i^2 \left(1 - \sum_{j:j<i \text{ and } a_j \geq a_i} \frac{p_j}{s_{j,i}^2} - (\frac{\pi^2}{6} - 1) \right) = a_i^2 \left(2 - \frac{\pi^2}{6} - \sum_{j:j<i \text{ and } a_j \geq a_i} \frac{p_j}{s_{j,i}^2} \right) \tag{22}$$

It is important that $\frac{\pi^2}{6} < 2$. For $i = 1, 2, \dots, n-2$ define

$$T_i := \sum_{j>i \text{ and } a_j \leq a_i} \frac{a_j^2}{s_{i,j}^2}. \tag{23}$$

Using Equations 11, 13, 22, and 23 we obtain:

$$R \geq L(2 - \frac{\pi^2}{6}) - \sum_{i=1}^{n-2} p_i \cdot T_i. \tag{24}$$

Using Claim 4 it follows that for all i

$$\sum_{j>i \text{ and } a_j \leq a_i} \frac{a_j^2}{s_{i,j}^2} \leq \sum_{j>i \text{ and } a_j \leq a_i} \frac{a_j^2}{(\sum_{j:j>i \text{ and } a_j \leq a_i} a_j)^2} \leq \frac{\pi^2}{6} - 1, \tag{25}$$

where again the $\frac{a_i^2}{a_i^2}$ term is missing from the summation and we get the -1. Thus from Equation 24 we obtain:

$$R \geq L(2 - \frac{\pi^2}{6}) - \sum_{i=1}^{n-2} p_i(\frac{\pi^2}{6} - 1) \geq L(2 - \frac{\pi^2}{6}) - R(\frac{\pi^2}{6} - 1). \tag{26}$$

It follow that

$$L \leq \frac{\frac{\pi^2}{6}}{2-\frac{\pi^2}{6}} R.$$

completing the proof of Theorem 3. ■

For the line case with the source as the leftmost vertex we believe that $c = \frac{\pi^2}{6}$ is the right constant. The instance with $a_i = 1$ for all i has the ratio of optimum without hitchhiking to the optimum with hitchhiking converging to $\frac{\pi^2}{6}$ as n tends to ∞.

When $\kappa > 2$, the same proof as above holds, where in Claim 4 one uses the limit of $\sum_{i=1}^{\infty} \frac{1}{i^\kappa}$, denoted by l, instead of $\frac{\pi^2}{6}$. We have $1 < l < 2$, which is all what the proof needs and in fact we obtain the smaller constant $\frac{l}{2-l}$ (since $l < \frac{\pi^2}{6}$). The proof of Claim 4 becomes more difficult as Taylor expansions are used instead of the current simple (although tedious) algebraic manipulations.

5 NP-Hardness and approximation

In this section we prove that MBH is NP-Hard. Agarwal et. al [1] claimed this result when $\gamma_{ack} = \gamma_p$ by a trivial reduction from Min-Energy Broadcast. Our version of MBH has $\gamma_{ack} = 0$ and thus allows arbitrarily small fractions $q(uv)$. In fact, the proof below works for any ratio γ_{ack}/γ_p.

As for approximation algorithms, one would apply the existing algorithm for Min-Energy Broadcast (without hitch-hiking). For arbitrary $c(uv)$ [3] and [5] give $O(\log n)$ approximation algorithm for Min-Energy Broadcast. Combined with Theorem 1, we obtain a $O(\log^3 n)$ approximation algorithm. For the Euclidean case, constant-ratio approximation algorithms are known [10], and thus we obtain a $O(\log^2 n)$ approximation algorithm. For the line cased, we presented in Section 3 a 2-approximation algorithm. The main result of this section is:

Theorem 5 *Min-Energy Broadcast with Hitch-Hiking is NP-Hard, even in the case where the power requirements are symmetric (when $c(uv) = c(vu)$ for all $u, v \in V$).*

Proof: We reduce from SAT, with an illustration in Figure 2. For every variable x_i we add six vertices, labeled $x_1, x_1', \overline{x}_1, \overline{x}_1', u_i, v_i$. For every clause C_j we add a vertex labeled C_j. We add one extra vertex s. The power requirements are symmetric and are either 1 or ∞. Below we specify the edges with power requirement 1. For all i, the edges sx_i', $s\overline{x}_i'$, $x_i'x_i$, $\overline{x}_i'\overline{x}_i$, $x_i\overline{x}_i$, $x_i'u_i$, $\overline{x}_i'$, u_i, x_iv_i, $\overline{x}_iv_i$ have power requirement 1. If clauses C_j contains literal z, we add an edge of power requirement 1 between the vertices labeled by C_j and z. All other $c(uv)$ are set to ∞. We show that the formula is satisfiable if and only if the MBH instance has a feasible solution of total power at most $2n + 1$.

Suppose first that the formula is satisfiable by the truth assignment Φ. We construct a solution to MBH of total power $2n + 1$, where n is the number of terminals. We pick τ to start with s, and then for every i, x_i' and x_i if x_i is true in Φ, or $\overline{x}_i'$ and $\overline{x}_i$ if x_i is false in Φ. The remaining vertices appear in arbitrary order. We set $p(s) = 1$. If x_i is true in Φ, we set $p(x_i') = p(x_i) = 1$, and if x_i is false in Φ, we set $p(\overline{x}_i') = p(\overline{x}_i) = 1$. For all the other vertices v we set $p(v) = 0$. One can check that this τ and power assignment yields a feasible solution of total power $2n + 1$ which does not use hitch-hiking in the sense that $q(uv)$ is either 0 or 1 for all pairs uv.

Now assume we have a permutation τ and a feasible power assignment of total power $2n + 1$. First we note that s must have $p(s) = 1$ just to reach other nodes. For every i, we have $p(x_i') + p(\overline{x}_i') \geq 1$ as the broadcast must reach u_i. Also $p(x_i) + p(\overline{x}_i) \geq 1$ since the broadcast must reach v_i. Since the total power is $2n + 1$, we deduce that $p(x_i') + p(\overline{x}_i') = 1$ and $p(x_i) + p(\overline{x}_i) = 1$.

Next we claim that either $p(x_i') = 1$ or $p(\overline{x}_i') = 1$. Indeed, assume $0 < p(x_i') < 1$ and therefore $0 < p(\overline{x}_i') = 1 - p(x_i') < 1$. One of x_i or $\overline{x}_i$ appears before the other in the permutation τ; there are two

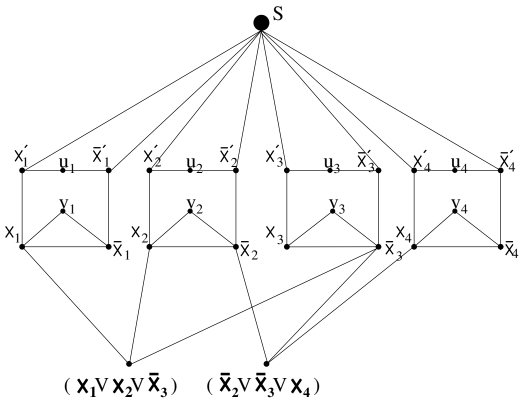

Figure 2: An illustration of the reduction, where the SAT instance has variables x_1, x_2, x_3, x_4.

symmetric cases and we pick x_i as the first one. Since x_i can only receive packets from x'_i (whose power is not enough), $\overline{x}_i$ (but $\overline{x}_i$ follows x_i in the permutation), v_i and the clauses C_j containing x_i as a literal, we deduce that either some v_i or some C_j has non-zero power. However, this would result in a total power exceeding $2n + 1$.

If $p(x'_i) = 1$, then $p(\overline{x}'_i) = 0$ and also $p(x_i) = 1$ since there is no way to reach $\overline{x}_i$ otherwise without exceeding total power $2n + 1$. Similarly, if $p(\overline{x}'_i) = 1$, then also $p(\overline{x}_i) = 1$. In the formula, we set x_i true if $p(x_i) = 1$ and 0 otherwise. As any vertex C_j receives the message, there must be a literal of C_j whose corresponding vertex has non-zero power. This vertex has power 1 and therefore the literal is true, satisfying the clause C_j. This argument holds for all clauses, and therefore the formula is satisfiable. ■

6 Conclusions

All the results presented in this paper hold for multicast as well, except that Min-Energy Multicast in the case of arbitrary power requirements matrix $c(uv)$ is harder than Min-Energy Broadcast in the same case - resulting in a worse approximation ratio for Min-Energy Multicast with Hitchhiking. Precisely, Min-Energy Multicast in the case of arbitrary power requirements matrix $c(uv)$ is equivalent to Directed Steiner Tree. Best known approximation ratio for Directed Steiner Tree is $O(n^\epsilon)$ for any $\epsilon > 0$ [6] and finding a poly-logarithmic approximation ratio remains a major open problem in approximation algorithms [21]. The recent paper of Halperin and Krautgamer [12] shows that Directed Steiner Tree admits no $\log^{2-\epsilon} n$ approximation unless NP has quasi-polynomial Las Vegas algorithms.

We left open the ratio of the optimum of MBH to the optimum of Min-Energy Broadcast in the Euclidean case. In the line case with the source at the leftmost point, we believe a ratio of $\frac{\pi^2}{6}$ holds - we know that this would be tight. Also, solving exactly MBH in the line case is open.

It would be interesting to find direct approximation algorithms for MBH. Our reduction can (with extra work) show that there is an $\varepsilon > 0$ such that approximation ratio less than $1 + \varepsilon$ is not possible unless $P = NP$. However, it does not rule out constant ratio approximation algorithms. For Min-Energy (without hitch-hiking) it is known that polynomial time algorithms with approximation ratio $\Omega(\log n)$ would imply $NP \subseteq DTIME(n^{\log \log n})$, even if the power requirements matrix must be symmetric (this follows from

[9] and a known reduction).

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