Homework Assignment 10
CS 330 Discrete Structures
Summer Semester, 2016

Name:

Due: 8:50am June 29 Wednesday, 2016

1. (40pts) Determine, with proof, whether each of the following language is regular:

   (a) Odd numbers, written in binary, reading most significant bit first.
   The following FSM recognizes this language, therefore it is regular.

   (b) Numbers not divisible by 3, written in binary, reading most significant bit first.
   The following FSM recognizes this language, therefore it is regular.
(c) The set of strings of zeros and ones containing the same number of zeros and ones.
This language is not regular by contradiction. If $L$ were regular, then because the intersection of regular languages is regular, $L \cap 0^*1^*$ would be regular; but this intersection is just $\{0^n1^n | n \geq 0\}$ which we know to be non-regular (Lecture note 10).

(d) The set $\{0^n1^m | n > 2m\}$.
This language is not regular. We can prove it by using the pumping lemma. Suppose it were regular and accepted by an FSM with $k$ states. Consider the action of the FSM on a string $0^{2k+1}1^k$. The FSM goes through $k + 1$ states as it reads the $k$ 1s, so it must repeat a state; the portion of the input between these two instances of the same state can be repeated as many times as we want, fooling the FSM into accepting a string that it should reject.

2. (40pts) Give a finite state machine that recognizes the following language.

The set $\{0^n1^m | n$ and $m$ are congruent modulo 3\}

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\begin{figure}
\centering
\begin{tikzpicture}
\node[state,initial] (0) at (0,0) {0};
\node[state] (1) at (1,1) {0,1};
\node[state] (2) at (2,1) {0,2};
\node[state] (3) at (4,1) {0,0};
\node[state] (4) at (5,1) {0};
\node[state] (5) at (0,-2) {1};
\node[state] (6) at (1,-2) {1,1};
\node[state] (7) at (2,-2) {1,2};
\node[state] (8) at (4,-2) {1,0};
\node[state] (9) at (5,-2) {dead};
\node[state] (10) at (2,-4) {2};
\node[state] (11) at (3,-4) {2,1};
\node[state] (12) at (4,-4) {2,2};
\node[state] (13) at (5,-4) {2,0};
\draw (0) edge[bend right] node[above] {1} (1);
\draw (1) edge[bend right] node[above] {1} (2);
\draw (2) edge[bend right] node[above] {1} (3);
\draw (3) edge[bend right] node[above] {0} (4);
\draw (4) edge[bend right] node[above] {0} (5);
\draw (5) edge[bend right] node[above] {0} (6);
\draw (6) edge[bend right] node[above] {0} (7);
\draw (7) edge[bend right] node[above] {0} (8);
\draw (8) edge[bend right] node[above] {0} (9);
\draw (9) edge[bend right] node[above] {0} (10);
\draw (10) edge[bend right] node[above] {0} (11);
\draw (11) edge[bend right] node[above] {0} (12);
\draw (12) edge[bend right] node[above] {0} (13);
\draw (13) edge[bend right] node[above] {0} (12);
\draw (12) edge[bend right] node[above] {0} (11);
\draw (11) edge[bend right] node[above] {0} (10);
\end{tikzpicture}
\end{figure}
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- Node with single digit $x$ denotes that number of 0s is $x \mod 3$
- Node with two digits $x, y$ denotes that number of 0s is $x \mod 3$ and number of 1s is $y \mod 3$

3. (40pts) Determine whether 1011 belongs to the following regular sets:

(a) $10^*1^*$: Yes.
(b) $0^*(10 \cup 11)^*$: Yes.
(c) $1(01)^*1^*$: Yes.
(d) $1^*01(0 \cup 1)$: Yes.
(e) $(10)^*(11)^*$: Yes.
(f) $1(00)^*(11)^*$: No.
(g) $(10)^*1011$: Yes.
(h) $(1 \cup 00)(01 \cup 0)^1^*$: Yes.