1. (20pts) Describe how to use the idea of DFS algorithm to determine if a given undirected graph is 2-colorable. You do not need to write a pseudo-code. Elaboration in English is sufficient.

   First, color the starting node with color 1, and every time DFS proceeds to the next node, switch the color. Then, the visited node is colored with color 2, and then DFS goes on by switching the color every time a new node is visited. During the traversal, if one node and an already-visited neighbor has the same color, then the graph is not 2-colorable.

2. (20pts) Give an example and show that Dijkstra’s algorithm may not work correctly if there is one or more negative-weight edges in the graph.
3. (20pts) Prove or disprove:

If the weights of all edges are distinct, the shortest path between any two vertices in a weighted graph is unique.

One simple counter example is:

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1   2   3   4
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4. (15pts) In unweighted graphs, the **distance** between two vertices \(u, v\) is defined as the length of the shortest path between \(u, v\), and the **diameter** is defined as the maximum distance between two distinct vertices in the graph. Find the diameters of \(K_5, K_{3,4}\) and a circle of 6 nodes.

(a) In \(K_5\), the maximum distance is only 1 because every node is connected to each other by one edge, therefore the distance is 1.

(b) In \(K_{3,4}\), the maximum distance is 2 because there exists an edge between any pair of \((u, v)\) where \(u \in V_1, v \in V_2\). Therefore, the maximum distance exists between two nodes within the same group. The distance is 2.

(c) In a circle of 6 nodes, the maximum distance is 3, therefore the diameter is 3.
5. (10pts) Find the optimum (with the minimum total weight) travelling salesman tour in the following graph.

Among all the tours, \((a, d, b, c, a)\) has the shortest total distance which is 17.