Illinois Institute of Technology
Department of Computer Science

First Examination

CS 330 Discrete Mathematics
Fall, 2009

11:25am–12:40pm, Wednesday, September 23, 2009
113 Stuart Building

Print your name and student ID, *neatly* in the space provided below; print your name at the upper right corner of *every* page. Please print legibly.

Name:	
Student ID:	

This is an *open book* exam. You are permitted to use the textbook, any class handouts, anything posted on the web page, any of your own assignments, and anything in your own handwriting. Foreign students may use a dictionary. *Nothing else is permitted*: No calculators, laptops, cell phones, Ipods, PDAs, communicators, etc.!

Do all five problems in this booklet. *All problems are equally weighted, so do not spend too much time on any one question.*

Show your work! You will not get partial credit if the grader cannot figure out how you arrived at an incorrect answer.

Question	Points	Score	Grader
1	20		
2	20		
3	20		
4	20		
5	20		
Total	100		

1. **Mathematical Induction.**

   Prove by induction that $\left(\frac{n}{3}\right)^n < n! < \left(\frac{n}{2}\right)^n$ if $n \geq 6$. You may assume that $2 < \left(1 + \frac{1}{n}\right)^n < 3$ for all $n \geq 0$. (*Hint*: Do separate inductive proofs for the upper and lower bounds.)

2. **Growth rates.**

   (a) Is $\binom{2n}{n} \in O(2^n)$? Prove your answer.

   (b) Is $n/H_n \in O(n)$? Prove your answer.

3. **Rules of Sum Product.** Use the variation of the Rule of Product to prove that the number of combinations of n things taken k at a time is

$$\binom{n}{k} = \frac{n!}{k!(n-k)!}$$

by taking event E as the number of permutations of n items.

4. **Binomial Coefficients.**

   Find the coefficient of x^{31} in each of the following polynomials. You need not simplify powers, factorials, or binomial coefficients.

   (a) $(1 + 2x)^{100}$

   (b) $(1 - x)^{-5}$

   (c) $(1 - x^2 + x^4 - x^6 + \cdots)^7$

   (d) $(1 - 5x)^{2n+1}$

5. **Combinatorial Interpretation.**

Give a combinatorial interpretation of the identity

$$\left[\sum_{k=0}^{n} \binom{n}{k}\right]^2 = \sum_{k=0}^{2n} \binom{2n}{k} = 2^{2n} = 4^n.$$

(*Hint*: Consider the number of ways to select a subset of a set of n couples. Note that you must give combinatorial interpretations for all four expressions.)