Illinois Institute of Technology
Department of Computer Science

Exam 1 (100 minutes)
CS 330 Discrete Structures
Summer Semester, 2016

Name:
A-ID:

- Only a pen and whatever provided by the instructor are permitted throughout the exam.
- You have to show your work. You will not get partial credits if the grader cannot figure out how you arrived at the answer.
- You do NOT need to calculate and simplify anything. Leave the notations as they are.

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1. Use a truth table to show that \((p \rightarrow q) \lor (p \rightarrow r)\) and \(p \rightarrow (q \lor r)\) are logically equivalent.
2. Use quantifiers, logical connectives, and predicates to express the following statements with a proposition, and then express the negation in simple English. (Do not simply say “It is not the case that ... ”).

- Every student in CS330 has taken at least one class from MATH.
  - Domain of $x$: all students.
  - $P(x)$: a student $x$ is in CS330.
  - $Q(x)$: a student $x$ has taken at least one class from MATH.

Proposition:

Negated sentence:
• No one has climbed every mountain in the Himalayas.
  – Define the predicates by yourself. (It doesn’t have to be sophisticated)

Defined predicates:

Proposition:

Negated sentence:
3. Use a direct proof to show that the product of two odd numbers is odd.
   (Hint: A number is odd if and only if it has a form $2a + 1$ for some integer $a$.)
• $n$ is a positive integer, you need to prove:

$$n \text{ is even if and only if } 7n + 4 \text{ is even.}$$

Prove the following two statements by contradiction in order to prove that proposition. (Hint: You need to assume the entire statement as false in proof by contradiction.)

Prove ‘$n$ is even’ $\rightarrow$ ‘$7n + 4$ is even’:

Prove ‘$7n + 4$ is even’ $\rightarrow$ ‘$n$ is even’:
4. Prove by induction that

\[ \sum_{i=0}^{n} (i + 1) = \frac{(n + 1)(n + 2)}{2} \]
5. Prove or disprove: If \( A, B, C \) are sets, then \( A - (B \cap C) = (A - B) \cap (A - C) \).
6. • How many different license plates can be made if each license plate consists of three letters and three digits or four letters and two digits? Letters and digits are not necessarily next to each other, and there is no repeated letters or digits. For example, AB3CD4 and A1B2C3 are both valid license plates.

• How many different strings can be made using all the letters in the word ‘GOOGOL’?
7. What is the probability that a fair coin lands heads four times out of five flips?
Suppose everyone in this CS330 has two capital letters as their initials (For example, TJ for Taeho Jung). Also suppose that every initial is equally likely to appear. What is the probability that at least two of the students have the same initial in our class? Our class has 10 students.
8. A fair red die and a fair blue die are rolled. What is the expected value of the sum of the number on the red die plus three times the number on the blue die? What is the variance of it?
9. Two identical urns contain balls. One of them has 6 red balls and 3 blue balls. The other one has 5 red balls and 8 blue balls. An urn is chosen at random and two balls are drawn at random from this urn, without replacement.

- What is the probability that both balls are red?

- What is the probability that the second ball is red, given that the first ball is red?
10. It is still the same two urns with the same balls (1st contain 6 red balls and 3 blue balls, 2nd contains 5 red balls and 8 blue balls). An urn is chosen at random and one ball is drawn at random from this urn. If you only know the instructor chose one ball and that is red, what is the probability that the instructor had chosen the 1st urn with 6 red balls?