Exam 2 (100 minutes)
CS 330 Discrete Structures
Summer Semester, 2016

Name:
A-ID:

- Only a pen and whatever provided by the instructor are permitted throughout the exam.
- You have to show your work. You will not get partial credits if the grader cannot figure out how you arrived at the answer.
- You do NOT need to calculate and simplify anything. Leave the notations as they are.

<table>
<thead>
<tr>
<th>Question</th>
<th>Maximum Points</th>
<th>Earned Points</th>
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<td>Total</td>
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</table>
1. Describe the simplest format of Big-Theta notation for the following functions (Do NOT need to explain the reason).

- $3n^2 + 8n + 2$
- $\binom{n}{3}$
- $(\log \log n)^3 + 3(2 + \log n)$
- $(n + 2) \log(n^2)$
- $2^{2n} + 3^n$
2. \(A[1], A[2], \cdots, A[n]\) is an array of \(n\) integers.

```plaintext
1: for \(i \leftarrow 1\) to \(n\) do
2:     if \(A[i] > 100\) then
3:         Break
4:     end if
5: end for
6: return \(i\)
```

- Analyze the time complexity of the algorithm in terms of the Big-Oh or Big-Theta notation.

- What does this algorithm do?
3. Fill in the blanks in the following table. Growth rates should be given in the simplest Big-Theta format. For the recurrence and closed formula, any one example is sufficient.

<table>
<thead>
<tr>
<th>Annihilator</th>
<th>Growth Rate</th>
<th>Recurrence</th>
<th>Closed formula</th>
</tr>
</thead>
<tbody>
<tr>
<td>((E - 4)^2)</td>
<td>(\Theta(4^n))</td>
<td>(S_n = S_{n-2} + n^3)</td>
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<td>(T(n) = 2T\left(\frac{n}{2}\right) + n)</td>
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</table>
4. The instructor presented an algorithm whose time complexity is defined by

\[ T(n) = 4T\left(\frac{n}{2}\right) + n \]

Later, a student said his algorithm is better than that, and the time complexity of that algorithm is defined by

\[ T(n) = 10T\left(\frac{n}{4}\right) + n \]

Is this student’s algorithm better than the instructor’s one? Explain your answer with Master Theorem.
5. Sort the following characters by using the 'merge sort', and show its procedure step by step.

\[ H, B, G, D, A, E, F, C \]
6. • What is the adjacency matrix of the following graph?

• What is the length of the path \((a, b, c, d, f, e)\)? (Note that the graph is weighted). What would be the length of the same path if the graph were unweighted?
7. • Is there an Euler circuit in the following graph? Why?

- Is the following graph planar? If so, draw a planar version. Otherwise, explain your answer.
8. Show how to color the following graph with 4 colors in the graph coloring problem.
9. Given the following graph:

- Show a result of bread-first search performed on the graph when it started at vertex $a$. Mark the distances for all vertices.
• Show a result of depth-first search performed on the graph when it started at vertex a. Mark the start/finish time for all vertices.