Print your name and student ID, *neatly* in the space provided below; print your name at the upper right corner of *every* page. Please print legibly.

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This is an *open book* exam. You are permitted to use the textbook (hardcopy only), any class handouts, anything posted on the web page, any of your own assignments, and anything in your own handwriting. Foreign students may use a dictionary. *Nothing else is permitted:* No calculators, laptops, cell phones, Internet-enabled watches, Ipads, Ipods, communicators, GPSes, etc.!

Do all five problems in this booklet. *All problems are equally weighted, so do not spend too much time on any one question.*

*Show your work!* You will not get partial credit if the grader cannot figure out how you arrived at your answer.

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1. Mathematical Induction.

Use induction to prove that

\[ \sum_{i=1}^{n} \frac{1}{i^2} < 2 - \frac{1}{n} \]

for all integers \( n > 1 \).
2. Growth rates.

(a) Does \( \binom{2n}{4} \) grow slower than, the same as, or faster than \( n^3 \)?

(b) Is \( n^{2^n} = O(n^2) \)?

In both cases, prove your answer.
3. **Rules of Sum and Product.**

Tom Martin designed the following myriorama for the Victoria & Albert Museum in London:

![Myriorama Image]

Each of the 16 small squares can be placed in 4 orientations and they can be arranged in any way in the $4 \times 4$ array. How many $4 \times 4$ arrangements can be formed with this myriorama? Remember, rotating an arrangement of the $4 \times 4$ array does not give you a new arrangement, but rotating an individual tile within the array does.

Justify your answer.
4. **Evaluation of Polynomials.**

You are given the polynomial

\[ 1 + x + x^2 + x^3 + \cdots + x^n \]

and the value of \( x \).

(a) Give an algorithm using only additions and multiplications to evaluate the polynomial, analyzing the number of each operation needed.

(b) If division is permitted, can you do better? Justify your answer.
5. **Binomial Coefficients.**

Derive the coefficient of $x^{10}$ in each of the following polynomials. You need not simplify powers, factorials, or binomial coefficients.

(a) $(1 - x)^{15}$
(b) $(1 - 3x)^k$
(c) $(1 - x^3 + x^6 - x^9 + \cdots)^2$
(d) $(1 + 5x)^{-n}$