CS330 Homework 6 * – version 2017-03-22

Questions
1. 4 points:
   a. Find a recurrence relation for the number of bit strings of length \( n \) that contain three consecutive 0s.
   b. What are the initial conditions?
   c. How many bit strings of length seven contain three consecutive 0s?

2. 4 points: A bus driver pays all tolls using only nickels and dimes, by throwing one coin at a time into the mechanical toll collector.
   a. Find a recurrence relation for the number of different ways the bus driver can pay a toll of \( n \) cents (where the order in which the coins are used matters).
   b. In how many different ways can the driver pay a toll of 45 cents?

3. 4 points: Suppose that there are \( n = 2^k \) teams in an elimination tournament, where there are \( n/2 \) games in the first round, with the \( n/2 = 2^{k-1} \) winners playing in the second round, and so on. Develop a recurrence relation for the number of rounds in the tournament.

4. 4 points: Solve the recurrence relation in number 3.

5. 4 points: Suppose that each person in a group of \( n \) people votes for exactly two people from a slate of candidates to fill two positions on a committee. The top two finishers both win positions as long as each receives more than \( n/2 \) votes.
   a. Write out (in pseudocode or words) a divide-and-conquer algorithm that determines whether the two candidates who received the most votes each received at least \( n/2 \) votes and, if so, determine who these two candidates are.
   b. Use the master theorem to give an \( O(…) \) estimate for the number of comparisons needed by the algorithm you devised in part (a).

   Hint: Assume that \( n \) is even and split the sequence of votes into two sequences, each with \( n/2 \) elements. A candidate cannot receive a majority of votes without receiving a majority of votes in at least one of the two halves.

6. 8 points: Find the solution to each of these recurrence relations with the given initial conditions. Use an iterative approach.
   a. \( a_n = -a_{n-1}, a_0 = 5 \)
   b. \( a_n = a_{n-1} + 3, a_0 = 1 \)
   c. \( a_n = a_{n-1} - n, a_0 = 4 \)
   d. \( a_n = 2a_{n-1} - 3, a_0 = -1 \)

7. 9 points: Solve these recurrence relations together with the initial conditions given.
   a. \( a_n = a_{n-1} + 6a_{n-2} \) for \( n \geq 2, a_0 = 3, a_1 = 6 \)
   b. \( a_n = 7a_{n-1} - 10a_{n-2} \) for \( n \geq 2, a_0 = 2, a_1 = 1 \)
   c. \( a_n = 6a_{n-1} - 8a_{n-2} \) for \( n \geq 2, a_0 = 4, a_1 = 10 \) [fixed 3/22]

8. 3 points: A model for the number of lobsters caught per year is based on the assumption that the number of lobsters caught in a year is the average of the number caught in the two previous years.
   a. Find a recurrence relation for \( \{L_n\} \), where \( L_n \) is the number of lobsters caught in year \( n \), under the assumption for this model.
   b. Find \( L_n \) if 100,000 lobsters were caught in year 1 and 300,000 were caught in year 2.