CS330 Homework 9

Questions

1. Points = 4. What kind of graph can be used to model a highway system between major cities where
   a. There is an edge between the vertices representing cities if there is an interstate highway between them?
   b. There is an edge between the vertices representing cities for each interstate highway between them?

2. Points = 2. (a) Explain how graphs can be used to model electronic mail messages in a network. Should the edges be
   directed or undirected? Should multiple edges be allowed? Should loops be allowed? (b) Describe a graph that models the
   electronic mail sent in a network in a particular week.

3. Points = 2. List the in-degrees and out-degrees of each vertex for the graph below. Give the sums of the in-degrees and the
   out-degrees (they should be equal and also equal to the number of edges in the graph).

4. Points = 2. In a Hollywood graph, actors are vertices; an edge between actors indicates they’ve been in one or more movies
   together. What does the degree of a vertex in the Hollywood graph represent? What does the neighborhood of a vertex
   represent? What do the isolated and pendant† vertices represent?

5. Points = 2. Is this graph bipartite? If it is, give a partition.

6. Points = 2. Is 6, 5, 4, 3, 2, 1 a possible degree sequence for a simple graph? If it is, draw the graph.

7. Points = 8.
   a. Draw a directed graph for this adjacency matrix:
   b. Give an adjacency matrix representation for this multigraph:

     \[
     \begin{pmatrix}
     0 & 1 & 0 \\
     1 & 0 & 1 \\
     0 & 1 & 0 \\
     \end{pmatrix}
     \]

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† A pendant vertex has a degree of 1.
c. Give an adjacency matrix for this graph

d. Draw a directed graph with the adjacency matrix below.

\[
\begin{bmatrix}
1 & 0 & 1 \\
0 & 0 & 1 \\
1 & 1 & 1
\end{bmatrix}
\]

8. Points = 3. What is the sum of the entries in a row of the adjacency matrix for an undirected graph? For a directed graph?

9. Points = 3. Are these graphs isomorphic? If yes, give a correspondence of the vertices. If not, give a rigorous argument why not.

10. Points = 3. Repeat, for these two graphs.

11. Points = 3. In a directed graph, a strongly-connected component is a subgraph where for every pair of vertices \( v \) and \( w \), there is a directed path from \( v \) to \( w \) and another from \( w \) to \( v \). The subgraph should be maximal (you can’t add another vertex without losing strong connectedness). Find the strongly-connected components of the following graph.


   a. Give its adjacency matrix representation, call it \( A \).

   b. Calculate and give \( A^2 \). What is the value of \( A^2[c, d] \); what does it value represent?

   c. Repeat for \( A^3 \) and \( A^3[c, d] \).