**CS330 Lecture 3 Activities**

**1.7 Quantified Statements**
- Frequently we need to negate quantified statements involving a universal or existential quantifiers.
- Recall two quantified statements are logically equivalent if they have the same truth value regardless of the domain of discourse, and regardless of the particular predicate(s) (represented by the predicate variable(s)).

**1.8 De Morgan's Laws - Key Concepts**
- The two De Morgan's laws for quantified statements are:
  - \[ \neg \forall x P(x) \equiv \exists x \neg P(x) \]: If it is not the case that \( P(x) \) is true for all \( x \), then there exists a counterexample \( x \) with \( P(x) \) false (and conversely).
  - \[ \neg \exists x P(x) \equiv \forall x \neg P(x) \]: If it is not the case that there exists an \( x \) for which \( P(x) \) is true, then for all \( x \), \( P(x) \) is false and conversely).

1. Consider the quantified statement \( \forall x J(x) \): “Every student in your class has taken a course in Java.”
   a. Use \{Alex, Morgan, Sam\} as the domain for \( x \). Rewrite the statement using conjunctions without a quantifier, and negate the result, and apply DeMorgan's law for negated conjunctions.

   b. Use an infinite domain for \( x \) and negate using DeMorgan's Law for the quantifier \( \forall x \).

2. Repeat Question 1 on \( \exists x J(x) \): “There exists a student in your class who has taken a course in Java.”

**1.9 Nested Quantifiers - Key Concepts**
- Quantifiers can be nested, with multiple quantifiers turning a multiple-variable predicate into a proposition.
- Nested-quantifier statements can be translated back and forth to expressive English sentences.
- Nested-quantifier statements can be negated through a methodical outside-to-inside process, using DeMorgan's Laws for negating quantifiers (\( \neg \forall x P(x) \equiv \exists x \neg P(x) \) and \( \neg \exists x P(x) \equiv \forall x \neg P(x) \))
- Doubly-nested quantifiers. A statement such as “Every real number has an additive inverse” translates to a nested-quantifier statement \( \forall x \exists y (x + y = 0) \), which itself can be thought of as \( \forall x \exists y P(x, y) \), or \( \forall x Q(x) \), where \( Q(x) \) is the predicate \( \exists y P(x, y) \).

---

* Some material from MATH230 Dr. Robert Ellis IIT-AMAT, zyBooks Disc. Math., and Disc. Math. and its Appl., Rosen (7th Ed.) is included under the academic/nonprofit Fair Use exception of US Copyright Law. This document must NOT be distributed beyond participants in IIT’s CS330. Table 1.4.4. is from the Discrete Mathematics zyBook.
3. If the domain of $x$ is finite and we can write a program to test $P(x)$, we can write a program to test for $\exists x P(x)$:
   Loop through the possible values of $x$, checking $P(x)$ as we go; if we find any $P(x)$ true, stop immediately and return true for $\exists x P(x)$. Return false if we run out of $x$ values.

   a. Describe how we can check for $\forall x P(x)$. [Hint: $\forall x P(x) \equiv \neg \exists x \neg P(x)$]

   b. Repeat, for $\exists x \exists y P(x, y)$

   c. Repeat, for $\forall x \exists y P(x, y)$

   d. Repeat, for $\exists x \forall y P(x, y)$

Order of Quantifiers is important
- $\forall x \forall y P(x, y)$ and $\forall y \forall x P(x, y)$ have the same truth value; so do $\exists x \exists y P(x, y)$ and $\exists y \exists x P(x, y)$
- But $\exists y \forall x P(x, y)$ and $\forall x \exists y P(x, y)$ don't always have the same truth value.
  - In general, $\exists y \forall x P(x, y)$ implies $\forall x \exists y P(x, y)$ but the converse may not hold.

2. Suppose we have the predicate $A(x, y) : "x \text{ is angry with } y"$ and the domain of discourse for both $x$ and $y$ are people at a party. How do you represent with quantifiers the two following English sentences:
   a. “There is a person at the party who is angry at some person at the party.” Include the possibility of people being angry at themselves.

   b. Repeat part (a) but this time exclude people angry at themselves.

3. Define $C(x): "x \text{ has a computer,}"$ and $F(x, y): "x \text{ and } y \text{ are friends}"$ where $x$ and $y$ are over the domain of students.
   a. Translate the statement $\forall x (C(x) \lor \exists y (C(y) \land F(x, y)))$ into English.
b. Translate the statement $\exists x \forall y \forall z ((F(x, y) \land F(x, z) \land (y \neq z)) \rightarrow F(y, z))$.

4. Define $B(x, y)$: “$x$ and $y$ are brothers,” $S(x, y)$: “$x$ and $y$ are siblings,” $L(x, y)$: “$x$ loves $y$.”

Translate the following into quantified statements, with the domain of discourse being all people:

a. Brothers are siblings.

b. Being a sibling is a reciprocal relationship.

c. Everybody loves somebody.

d. There is someone who is loved by everyone.

e. There is someone who loves someone.

f. Everyone loves herself/himself.

5. Define the predicate $L(x, y)$: “$x$ likes $y$” where the domain of discourse for both $x$ and $y$ are students.

a. Translate the sentence “There is a student whom every other student likes” into quantified logic.
b. Write the English sentence that is the negation of “There is a student whom every other student likes”.

c. Use DeMorgan's Laws to negate the quantified statement you obtained in (a) so that there are no negation operators to the left of any quantifiers.

d. Translate the result of (c) back into an English sentence.