CS330 Activities for Lecture 12*

5.4 The Generalized Product Rule

- **Terminology:** We make a choice by selecting from a set of options. We can make a sequence of choices, where each choice is made from its set of options. (Note: zyBooks doesn't mention “options”.)
- In some counting situations the number of options for a choice remains the same, but the actual options may change.
- **Example:** How many ways are there to order Alex, Charlie, and Chris in line at the coffee shop to get espresso?
  - We have three choices: 1st person, 2nd person, 3rd person.
  - The 1st choice has 3 options: {Alex, Charlie, Chris}.
  - The 2nd choice has 2 options from whoever doesn't go 1st.
  - The 3rd choice has 1 option from whoever is left.
  - Total: 3 options × 2 options × 1 option = 6 orderings.

**Generalized product rule**

- Consider a set $S$ of sequences of $k$ items. Suppose:
  - There are $n_1$ choices for the first item.
  - For every possible choice for the first item, there are $n_2$ choices for the second item.
  - For every possible choice for the first and second items, there are $n_3$ choices for the third item.
  - ...
  - For every possible choice for the first $k$-1 items, there are $n_k$ choices for the $k$’th item
- Then the total number of possibilities is $n_1 \cdot n_2 \cdot \ldots \cdot n_k$.

**Questions**

1. An ice cream shop has 32 flavors of ice cream. In a triple cone, you get to pick 3 different flavors for your top scoop, middle scoop, and bottom scoop. The order of flavors is significant; e.g., chocolate / vanilla / strawberry is different from strawberry / vanilla / chocolate. How many distinct triple cones are possible?

2. You own 100 children's books, but opting for sanity, only allow your child to choose 3 for bedtime reading. The order is terribly important. How many possibilities are there?

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5.5 Counting Permutations

- **r-permutations from a set with n elements.** Let \( r \) and \( n \) be positive integers with \( r \leq n \). The number of \( r \)-permutations from a set with \( n \) elements is denoted by \( P(n, r) \).\

\[
P(n, r) = \frac{n!}{(n-r)!} = \frac{n \cdot (n-1) \cdot \ldots \cdot (n-r+1) \cdot (n-r+2) \ldots 2 \cdot 1}{(n-r) \cdot (n-r-1) \ldots 2 \cdot 1} = n \cdot (n-1) \cdot \ldots \cdot (n-r+1)
\]

- **Permutations of a finite set.** A finite set with \( n \) elements has \( P(n, n) = n! \) permutations.
- **Permutations with compound symbols.** Sometimes we construct a permutation on a set of symbols but require certain symbols to appear next to each other in a fixed order. For example, maybe every \( c \) must have a \( b \) to its immediate left. In this case we can treat \( b c \) as a single symbol in constructing the permutation.

Questions

3. You can fit 40 of 100 books on a single bookshelf. How many ways are there to choose and arrange the books (assuming all books side by side)? [Write a formula for the number; actually calculating it is hard.]

4. Four bridesmaids and four groomsmen are paired together for a wedding. How many ways are there to arrange them in a row for a photograph if every bridesmaid must be immediately to the left of her groomsman partner?

5. How many ways are there to arrange a bride and groom, 3 bridesmaids, and 3 groomsmen in a row for a photograph, if:
   - The bride must be to the left of the groom;
   - The bridesmaids must be to the left of the bride; and
   - The groomsmen must be to the right of the groom

\[\text{\dagger And } C(n, r) = \text{the number of ways to choose a subset of } r \text{ items from a set of } n \text{ items } = P(n, r) \times r!\]

6. How many ways are there to arrange 6 members of a wedding party in a row for a photograph if the bride must be to the left of the groom?

7. We have a poker player who sorts a 5-card hand by putting all the numbered cards to the right of all the face cards plus ace. However, the numbered cards don’t have to be in any particular order, nor do the face cards, so (e.g.) J K Q 10 7 and J Q K 7 10 count as different hands. How many different poker hands can we have?

8. The local diner is selling a Dagwood sandwich: You can arrange one slice each of ham, salami, roast beef, corned beef, and pepperoni, american cheese, and swiss cheese, plus, you have lettuce, tomato, onion, ketchup, mustard, and a pimento olive (on top of the sandwich, with a toothpick, to hold everything together). There’s a restriction: The ketchup must be somewhere under the lettuce and the mustard must be somewhere above the lettuce. (There can be items between the ketchup or mustard and the lettuce, as in, e.g. ketchup, tomato, lettuce.) How many different Dagwood sandwiches exist?