**CS330 Activities for Lecture 21**

**9.3 Graph Isomorphism**

- Graphs are isomorphic when they have the same structure: If you can rename the vertices of one graph to the same names as in the graph and get the same edges, the graphs are isomorphic.
- More formally, two graphs are isomorphic if there is a correspondence between the vertex sets of each graph such that there is an edge between two vertices of one graph if and only if there is an edge between the corresponding vertices of the second graph.

Some properties like total degree are “preserved under isomorphism”: If two graphs are isomorphic, they have the same total degree; more importantly, if two graphs have different total degrees, then they aren’t isomorphic.

A property is said to be preserved under isomorphism if whenever two graphs are isomorphic, one graph has the property if and only if the other graph also has the property.

Similarly, the degree sequence of a graph (a list of the degrees of all of the vertices in non-increasing order) is preserved under isomorphism.

**Questions**

1. (Exercise 9.3.1) Prove that the two graphs below are isomorphic.
2. (Exercise 9.3.3 b) Draw all non-isomorphic simple graphs with four vertices. Do not label the vertices of the graph. You should not include two graphs that are isomorphic. Remember that it is possible for a graph to appear to be disconnected into more than one piece or even have no edges at all.

9.4: Paths, Cycles, and Connectivity

- Given a graph, a walk in the graph is a sequence of vertices where each pair of vertices has an edge in the graph. Said another way, you get a walk by following a sequence of edges. You can visit the same vertex more than once, and you can traverse the same edge more than once (and in different directions, for an undirected graph). Since you can repeat edges, a walk can be any length (v₁ to v₂ to v₁ to v₂ to … ).
- A circuit is a walk that begins and ends at the same vertex. If a circuit consists of unique vertices (doesn’t visit a vertex more than once except for the first/last vertex), then it’s a cycle. If a walk consists of unique vertices, then it’s a path. (So a cycle is a path that’s a circuit.)

Questions

3. Study the graph below and name some examples of walks, circuits, cycles, and paths. How long are the longest walks, circuits, cycles, and paths, and give examples.

4. It’s always possible to turn a walk into a path by dropping any loops in the walk. Give an example, using the graph from the previous problem.

† Trick question