CS330 Recitation 1

Review additional exercises from the zyBook sections 1.1 - 1.6, the problems below, and any questions from HW #1.

1. [From Rosen] Let $p$ and $q$ be the propositions “Swimming in the New Jersey shore is allowed” and “Sharks have been spotted near the shore,” respectively. Express each of these compound propositions as an English sentence.
   a. $\neg q$
   b. $p \land q$
   c. $\neg p \lor q$
   d. $p \rightarrow \neg q$
   e. $\neg q \rightarrow \neg p$
   f. $\neg p \rightarrow \neg q$
   g. $p \leftrightarrow \neg q$
   h. $\neg p \land (p \lor \neg q)$

2. [From Rosen] Let $p$ be the proposition “It is below freezing” and let $q$ be the proposition “It is snowing”. Write the following propositions using $p$ and $q$ and logical connectives.
   a. It is below freezing and snowing.
   b. It is below freezing but not snowing.
   c. It is not below freezing and it is not snowing.
   d. It is either snowing or below freezing (or both).
   e. If it is below freezing, it is also snowing.
   f. It is either below freezing or it is snowing, but it is not snowing if it is below freezing.
   g. That it is below freezing is necessary and sufficient for it to be snowing.

3. [From Rosen] You can graduate only if you have completed the requirements of your major and you do not owe money to the university and you do not have an overdue library book. Translate this to a proposition using logical connectives and $g$: “You can graduate,” $m$: “You owe money to the university,” $r$: “You have completed the requirements of your major,” and $b$: “You have an overdue library book.”

4. [From Rosen] Are these system specifications consistent? “The system is in multiuser state if and only if it is operating normally. If the system is operating normally, the kernel is functioning. The kernel is not functioning or the system is in interrupt mode. If the system is not in multiuser state, then it is in interrupt mode. The system is not in interrupt mode.” [Hint: It may help to give proposition letter names to various statements, as in the previous problem.]

5. [From Rosen] Show that each implication below is a tautology, using truth tables.
   a. $(p \land q) \rightarrow p$
   b. $p \rightarrow (p \lor q)$
   c. $\neg p \rightarrow (p \rightarrow q)$
   d. $(p \lor q) \rightarrow (p \rightarrow q)$
   e. $\neg (p \rightarrow q) \rightarrow p$
   f. $\neg p \rightarrow (p \rightarrow q)$
   g. $\neg (p \rightarrow q) \rightarrow \neg q$

6. [From Rosen] Show that each implication from the previous exercise is a tautology, but without using truth tables. [Hint: Take each implication and transform it to True using the logical equivalence rules from Table 1.4.4 in the Discrete Mathematics zyBook.]

7. [From Rosen] Determine whether $(\neg q \land (p \rightarrow q)) \rightarrow \neg p$ is a tautology.

8. [From Rosen] Let $P(x)$ be the statement “$x$ spends more than five hours every weekday in class,” where the domain of discourse for $x$ consists of all students. Express each of these quantifications in English.
   a. $\exists x \ P(x)$
   b. $\forall x \ P(x)$
   c. $\exists x \ \neg P(x)$
   d. $\forall x \ \neg P(x)$

9. [From Rosen] Translate these statements into English where $C(x)$ is “$x$ is a comedian” and $F(x)$ is “$x$ is funny”. Assume the domain of discourse consists of all people.
   a. $\forall x \ (C(x) \rightarrow F(x))$
   b. $\forall x \ (C(x) \land F(x))$
   c. $\exists x \ (C(x) \rightarrow F(x))$
   d. $\forall x \ (C(x) \land F(x))$

10. [From Rosen] Find a counterexample, if possible, to these universally quantified statements, where the domain of discourse for all variables consists of all integers.
    a. $\forall x \ (x^2 \geq x)$
    b. $\forall x \ (x > 0 \lor x < 0)$
    c. $\forall x \ (x = 1)$

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