Review additional exercises from Zybook 2.1-2.2, the below problems, and any questions from HW#2.

1. Write a direct proof showing that the square of an even number is an even number

2. Write a direct proof showing that additive inverse (= “the negative) of an even number is an even number

3. Prove that over the integers\(^\dagger\) if \(m + n\) and \(n + p\) are both even integers, then \(m + p\) is also even.

4. Write a direct proof showing that the product of two odd integers is odd.

5. Write a direct proof showing that every odd integer is the difference of two squares.

6. Prove that if integer \(n\) is a perfect square (i.e., it equals the square of some integer), then \(n + 2\) is not a perfect square. (Note that since squares are \(\geq 0\), you only need to look at natural numbers for \(n\).)

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\(^\dagger\) “Over the integers” means “assume all the values that follow are integers.” (\(m, n, p\) here).