**CS330 Recitation 4**

**Part 1**
Review additional exercises from zyBook 2.3-2.4, the problems below, and any questions from HW 2.

1. Use a contrapositive proof to show that the square of an even number is also even.†
2. Use a contrapositive proof to show that if \( n \) is an even integer, then so is \( -n \).
3. Use a contrapositive proof to show that every odd integer is the difference of two squares.
4. Prove (using proof by contradiction) that for integer \( m \), \( n \), and \( p \), if \( m + n \) and \( n + p \) are even, then so is \( m + p \).
5. Prove (using proof by contradiction) that if \( n \) is a perfect square, then \( n + 2 \) is not a perfect square.

**Part 2**
Review additional exercises from zyBook 3.1-3.3, the problems below [from Rosen], and any questions from HW 3.

9. A *palindrome* is a string that reads the same forward and backward. Describe an algorithm for determining whether a string of \( n \) characters is a palindrome.

23. Describe an algorithm that determines whether a function from a finite set to another finite set is onto.

29. A *mode* of a list of integers is an element that occurs at least as often as each of the other elements. Devise an algorithm that finds a mode in a list of nondecreasing integers.

1. Determine whether each of these functions is \( O(x) \)
   a. \( f(x) = 10 \)
   b. \( f(x) = 3x+7 \)
   c. \( f(x) = x^2 + x + 1 \)
   d. \( f(x) = 5 \log x \)
   e. \( f(x) = \lfloor x/2 \rfloor \)
   f. \( f(x) = \lceil x/2 \rceil \)

7. Find the least integer \( n \) such that \( f(x) \) is \( O(x^n) \) for each of these functions:
   a. \( f(x) = 2x^3 + x^2 \log x + 1 \) [Hint: \( \log x \in O(x) \) but \( x \notin O(\log x) \)]
   b. \( f(x) = 3x^3 + (\log x)^6 \)
   c. \( f(x) = (x^4 + x^2 + 1) / (x^3 + 1) \)
   d. \( f(x) = (x^4 + 5 \log x) / (x^3 + 1) \)

9. Show that \( x^2 + 4x + 17 \in O(x^3) \) but that \( x^3 \notin O(x^2 + 4x + 17) \) [Hint: It's sufficient to show \( x^3 \notin O(x^2) \).]

3. Suppose that an element is known to be among the first 4 elements in a list of 32 elements. Would a linear search or a binary search locate this element more rapidly?