Last time → Lecture 1 - course mechanics

CS.illinois.edu/
ncs330

Logic - reason about properties
Propositional logic
- things are true or false

Proposition statements, judgements
- declarative stuff that is either true or false
Propositions - it's sunny now

\( t = \) is it 2:30 pm now?

- what time is it?  
- not prop.

Proposition

Variables

stand for propositions  \( T \) \( F \)

P191

atomic - can't be broken up into
nonatomic

smaller propositions
Combine propositions using connectives

- $\neg p$ negation “not $p$”
- $p \land q$ conjunction “$p$ and $q$” conjuncts
- $p \lor q$ disjunction “$p$ or $q$” inclusive or
- $p \oplus q$ exclusive or “$p$ xor $q$” disjuncts

(math, logic, etc.)

$T \lor T = T$ $p \lor q$ one or more of $p, q$ are true $T \lor F$ True

$T \land T = T$ $p \land q$ exactly one of $p, q$ are true

(english statements)

it is raining $\land$ we are having a picnic

$F \land F$ $T \land T$ $T \land F$ $F$
Truth tables - show the value of a connected prop. as a fun of its sub-props

$p | \neg p
T | F
F | T

$p, q | p \cdot q
F, F | F
F, T | F
T, F | F
T, T | T

$p, q | p \lor q
F, F | F
F, T | T
T, F | T
T, T | T

$p, q | p \oplus q
F, F | F
F, T | T
T, F | T
T, T | F
<table>
<thead>
<tr>
<th>p</th>
<th>q</th>
<th>r</th>
<th>p ∨ q ∨ r</th>
<th>normal English</th>
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Translate English to prop logic:

- \( p \) = "I like cats"
- \( q \) = "I like dogs"
- \( r \) = "I like mammals"

\( p \oplus q \) = I like cats or dogs (but not both)

\( pvq \) = I like cats or dogs or both
Tautology - all rows of a T-Table's column are T

Contradiction - all rows are F

Contingency - at least 1 T and 1 F

<table>
<thead>
<tr>
<th>P</th>
<th>P \land Q</th>
<th>P \lor Q</th>
<th>P \Rightarrow Q</th>
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Not taut = contr. or contingency
Not contr. = taut. or contingency

If \( p \) is a taut then \( \neg p \) is a contr.
If \( p \) is a contr. then \( \neg p \) is a taut.
Implication: \( p \rightarrow q \) conditional \[ \begin{array}{c|c|c}
 p & q & p \rightarrow q \\
 T & T & T \\
 T & F & F \\
 F & T & T \\
 F & F & T \\
 \end{array} \]

Absurdity implies anything.

If \( 1 + 1 = 1 \) then I'm Santa Claus

\[ \{ \text{me}\} \]

\[ \{ \text{s.c.}\} \]

\[ \{ \text{me}, \text{s.c.}\} \]