CS 330 class 3

Tue Jan 26

- Classes 01, 02 - on Panopto <= 8B
- Fuller schedule posted
- Note exam dates

Bit more propositional logic

- English "or" - prop. \lor
- English "if" - prop. \iff biconditional
- Prop. "logical equivalence"

\( p \equiv q \) means \( p \iff q \) is a tautology

- Can replace equals for equals
  - \( p \equiv q \text{ and } q \equiv r \text{ implies } p \equiv r \)
  - \( p \equiv q \text{ and } p \equiv r \text{ implies } q \equiv r \)
\[ F \land F = F \quad T \]

\[(p \iff q) \land (q \iff r) \iff (p \iff r)? \]

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No

see laws about propositions - table

Predicate logic
Add \( \{ \text{domain of values, variables, predicates} \} \) and quantifiers

defined
\[ \text{thing 1} \equiv \text{thing 2} \]
\[ p \iff q \equiv \neg p \lor q \]
Predicate - Statement that is true or false depending on values of variables

A predicate is like a function of values to T/F. \( P(x) \equiv x \text{ is a dog} \)

\[
P(x, y) \equiv x > y^2 \quad P(2, 1) \checkmark \\
P(3, 2) \checkmark \\
h(x) \checkmark \\
Q(n) \equiv n \text{ is prime} \land n < 100
\]
Quantifier - how many/which values make the predicate true?

Most important:

\( \forall \) - for all
\( \exists \) - there exists

\[ \forall x \in \mathbb{Z} \ (x \geq 0) \]
\[ \forall x \in \mathbb{Z} \ (x > 0) \]

\( \forall x \ P(x) \) - \( P(x) \) true for every \( x \) in domain

\( \exists x \ P(x) \) - \( P(x) \) true for at least one \( x \) in domain

\( \exists x \in \mathbb{Z} \ (x > 0) \) - witness value
\forall x \; P(x)

Loops!

while there is an unchecked \( x \) in domain

if \( P(x) \) false, exit loop with false for value of \( \forall x \; P(x) \)

get another \( x \)

use true for value of \( \forall x \; P(x) \)

\exists x \; P(x)

while there is an unchecked \( x \) in domain

if \( P(x) \) is true, exit loop with true

get another \( x \)

use false for \( \exists x \; P(x) \)
For finite domain, \( \forall \) is like \( \land \), \( \exists \) is like \( \lor \)

Say domain = \{1, 2, 3\}, \( P(x) \equiv x \) is prime

\[ \forall x \in \text{domain}, P(x) \equiv P(1) \land P(2) \land P(3) \equiv \text{T} \land \text{F} \land \text{T} = \text{F} \]

\[ \exists x \in \text{domain}, P(x) \equiv P(1) \lor P(2) \lor P(3) \equiv \text{F} \lor \text{T} \lor \text{T} = \text{T} \]

\[ R(n) \equiv n \equiv 0 \]

\[ \forall x \in \text{domain}, \exists x \in \text{domain} : P(x) \equiv P(1) \lor P(2) \lor P(3) \equiv \text{F} \lor \text{T} \lor \text{T} = \text{T} \]

Other quantifiers:

- \( \forall R(x) \): for exactly one \( x \)
- \( \exists R(x) \): for an infinite number of \( x \)
- \( \forall R(x) \): for a finite # of \( x \)
- \( \exists R(x) \): for a majority of \( x \) from finite domain
Unique existence: \( \exists! x \ P(x) \equiv \exists x \ P(x) \land \neg \exists y \ (P(y) \land y \neq x) \)

Domain: \( \mathbb{Z} \)

Domain: \( \mathbb{N} \)?

(\( \exists x \ P(x) \)) \land \_ \_ \_ \_ ?

Scope of quantifier:

Precedence:

\( \forall x \ P(x) \lor Q(x) \equiv (\forall x \ P(x)) \lor Q(x) \) ?

\( (\forall y \ P(y)) \lor Q(x) \) ?

More? Less?

\( \forall x \) bind \_ \_ \_ \_ tightly

than other operators ? \lor, \land \_ \_ \_ ?
Translating English to logic with quantifiers

"Every vegetable in the store is green"
∀x ∈ Veg (Green(x))

"Some vegetable in this store is not green"
∃x (∼Green(x))

\[
\begin{array}{c|c|c|c|c}
 x & m & \text{true} & \text{false} \\
 0 & 0 & F & T \\
 1 & 1 & F & T \\
\end{array}
\]

Nested Q's

∀x ∈ M (\exists m \left( m > x^2 \right))

x = 0, m = 1 witness (∀x) E
x = 1, m = 1 not witness
x = 2, \( m = 2 \) is ?

x = 2, \( m = 5 \)
Negating Quantified Predicates

\[ \forall x \, P(x) \equiv \text{for every vegetable in this store is green} \]

\[ \neg \forall x \, P(x) \equiv \text{it's not the case that every vegetable in this store is green} \]

\[ \equiv \exists x \, \neg P(x) \equiv \text{there is a vegetable in this store that is not green} \]

\[ \equiv \exists x \, \neg P(x) \]