Homework Assignment 3
CS 330 Discrete Structures
Fall Semester, 2017

Due: Friday, October 13, 2017

1. The Chinese calendar uses a cycle of 60 names for days (also for years, as is better-known). The name is formed by combining a celestial stem, tiān gān, with a terrestrial branch, dì zhī. The celestial stems,

(1) Jiā (6) Ji
(2) Yí (7) Géng
(3) Bìng (8) Xīn
(4) Dìng (9) Rén
(5) Wù (10) Guí

are untranslatable, though they are sometimes associated with the 5 elements (tree, fire, earth, metal, and water), each in its male and female form. The terrestrial branches

(1) Zǐ (Rat) (7) Wǔ (Horse)
(2) Chóu (Ox) (8) Wèi (Sheep)
(3) Yín (Tiger) (9) Shēn (Monkey)
(4) Mǎo (Hare) (10) Yǒu (Fowl)
(5) Chén (Dragon) (11) Xū (Dog)
(6) Sì (Snake) (12) Hái (Pig)

are also untranslatable; the English names—traditional animal totems—given for the 12 branches corresponding to the years of the Chinese “Zodiac” are not translations from the Chinese.

Names are assigned sequentially, running through the decimal and duodenary lists simultaneously: The first name is jiāzǐ, the second is yíchóu, the third is bìngyín, and so on. Because the least common multiple of 10 and 12 is 60, the cycle of names repeats after the sixtieth name, guíhái.

(a) How many people do you need to have the probability be at least 0.5 that some pair of them have the same Chinese celestial stem birthday?
(b) How many people do you need to have the probability be at least 0.5 that some pair of them have the same Chinese terrestrial branch birthday?
(c) How many people do you need to have the probability be at least 0.5 that some pair of them have the same Chinese day name birthday?

2. Page 468, problem 32
3. Page 477, problem 22
4. Page 493, problem 40
5. The students in CS 330 decide to send a representative to the president of IIT to complain about their terrible classroom. But nobody in the class will agree to be the representative, so they decide on the following strategy: Each student will toss his/her coin simultaneously; if all the coins but one show the same face, the person who tossed the odd coin must be the representative. If any other combination of heads and tails occurs, the class will toss coins again.

(a) Assuming that there are \( n \) students in the CS 330 and that all of the coins are fair, what is the expected number of rounds (number of coin tosses each student has to make)? (\textit{Hint}: Explain why the probability of an odd toss in any given round is \( p = n / 2^{n-1} \). Then explain why the probability of ending on the \( k \)th round is \( (1 - p)^{k-1} p \). Finally, using the method of Example 25 on page 167 of Rosen, get the answer \( 2^{n-1} / n \).)

(b) One of the \( n \) students decides to cheat and uses a false coin that comes up heads much more often than tails. How does this change the answer in part (a)? Explain.

6. (a) A couple decides to keep having children until they have both sexes. If \( p \) is the probability of having a boy, what is the expected number of children they will have? (\textit{Hint}: Use the method of Example 25 on page 167 of Rosen to evaluate the summation, giving the answer \( \frac{1-p+p^2}{p(1-p)} \). Does your answer make sense if \( p = 0 \) or \( p = 1 \)? Explain.

(b) A different couple decides to keep having children until they have a child of the same sex as their first child. If \( p \) is the probability of having a boy, \( 1 > p > 0 \), what is the expected number of children they will have? What if \( p = 0 \) or \( p = 1 \)? Explain.