1. What is the coefficient of $x^8y^9$ in the expansion of $(3x + 2y)^{17}$?

2. The row of Pascal’s triangle containing the binomial coefficients $\binom{10}{k}$, $0 \leq k \leq 10$, is:

$$1 \quad 10 \quad 45 \quad 120 \quad 210 \quad 252 \quad 210 \quad 120 \quad 45 \quad 10 \quad 1$$

Use Pascal’s Identity to produce the row immediately following this row in Pascal’s triangle.

3. Show that if $n$ is a positive integer, then

$$1 = \binom{n}{0} < \binom{n}{1} < \cdots < \binom{n}{\lfloor n/2 \rfloor} = \binom{n}{\lceil n/2 \rceil} > \cdots > \binom{n}{n-1} > \binom{n}{n} = 1$$

4. Prove the identity $\binom{n}{r} \binom{r}{k} = \binom{n}{k} \binom{n-k}{r-k}$, whenever $n, r, k$ are nonnegative integers with $r \leq n$ and $k \leq r$,

a) using a combinatorial argument.

b) using an argument based on the formula for the number of $r$-combinations of a set with $n$ elements.

5. An employee joined a company in 2019 with a starting salary of $60,000. Every year this employee receives a raise of $1,000 plus 5% of the salary of the previous year.

a) Set up a recurrence relation for the salary of this employee $n$ years after 2019.

b) What will the salary of this employee be in 2027?

c) Find an explicit formula for the salary of this employee $n$ years after 2019.

6. How many ways are there for 10 women and six men to stand in a line so that no two men stand next to each other? Hint: first position the women and then consider possible positions for the men.

7. a) Find a recurrence relation for the number of ternary strings that contain two consecutives 0s.

b) What are the initial conditions?

c) How many ternary strings of length six contain two consecutive 0s?