Illinois Institute of Technology
Department of Computer Science

First Examination

CS 330 Discrete Mathematics
Spring, 2008

10am–11:15am, Wednesday, February 27, 2008
121 Life Sciences

Print your name and student ID, *neatly* in the space provided below; print your name at the upper right corner of *every* page. Please print legibly.

Name:	
Student ID:	

This is an *open book* exam. You are permitted to use the textbook, any class handouts, anything posted on the web page, any of your own assignments, anything in your own handwriting, and a calculator. Foreign students may use a dictionary. *Nothing else is permitted*: No calculators, laptops, cell phones, Ipods, communicators, etc.!

Do all five problems in this booklet. *All problems are equally weighted, so do not spend too much time on any one question.*

Show your work! You will not get partial credit if the grader cannot figure out how you arrived at your answer.

Question	Points	Score	Grader
1	20		
2	20		
3	20		
4	20		
5	20		
Total	100		

1. **Mathematical Induction.**

   Prove by induction that for $n \geq 1$,

$$\sum_{k=1}^{n-1} k \times k! = n! - 1.$$

2. **Growth rates.**

    (a) Is $\binom{n}{k} \in O(2^n)$ for $k \leq 10$? Prove your answer.

    (b) Is $nH_n \in O(n)$? Prove your answer.

3. **Algorithms/Binomial Coefficients.**

(a) Suppose you compute $\binom{n}{i}$ with the recurrence

$$\binom{n}{i} = \binom{n-1}{i-1} + \binom{n-1}{i}$$

using the following recursive function:

```
FUNCTION Combination(n,i)
  BEGIN
    IF i=0 OR i=n THEN RETURN 1;
    ELSE RETURN Combination(n-1, i-1) + Combination(n-1, i);
  END
```

Analyze the number of additions needed to compute $\binom{n}{i}$.

(b) Show how to compute $\binom{n}{i}$ in $O(i)$ arithmetic operations (not necessarily additions)..

(c) Using (b), analyze the number of arithmetic operations used to compute the sum $\sum_{i=0}^{k} \binom{n}{i}$.

4. **More Binomial Coefficients.**

   Find the coefficient of x^{31} in each of the following polynomials.

   (a) $(1+x)^{25}$

   (b) $(1-x)^{-25}$

   (c) $(1-x^3+x^6-x^9+\cdots)^5$

   (d) $(1-3x)^{n+1}$

5. **Combinatorial Interpretation.**

   Give combinatorial interpretations of the identities

   (a) $n! = \binom{n}{k} k!(n-k)!$

   (b) $\sum_{k \geq 0} \binom{n}{2k} = 2^{n-1}$