

Illinois Institute of Technology
Department of Computer Science

Second Examination

CS 330 Discrete Mathematics
Spring, 2008

10am–11:15am, Friday, April 4, 2008
111 Stuart Building

Print your name and student ID, *neatly* in the space provided below; print your name at the upper right corner of *every* page. Please print legibly.

Name:
Student ID:

This is an *open book* exam. You are permitted to use the textbook, any class handouts, anything posted on the web page, any of your own assignments, and anything in your own handwriting. Foreign students may use a dictionary. *Nothing else is permitted*: No calculators, laptops, PDAs, cell phones, etc.

Do all five problems in this booklet. *All problems are equally weighted, so do not spend too much time on any one question.*

Show your work! You will not get partial credit if the grader cannot figure out how you arrived at your answer.

Question	Points	Score	Grader
1	20		
2	20		
3	20		
4	20		
5	20		
Total	100		

1. Coin Tossing

We have a coin for which the probability of tossing heads is p , and the probability of tossing tails is $1 - p$. We toss the coin for $n > 2$ times.

- (a) What is the probability that there were no heads?
- (b) What is the probability that there were at least one head?
- (c) What is the expected number of heads we will get?

2. Recurrences

Solve the following recurrences using annihilators, finding the general form of each solution. You do not need to solve the simultaneous linear equations that give the coefficients in the general form, but you must give those equations.

(a) $S(0) = 3, S(n) = 2S(n-1) + 2, n \geq 1.$

(b) $F(0) = 3, F(1) = 1, F(n) = 6F(n-1) - 9F(n-2), n \geq 2.$

(c) $a_0 = 1, a_1 = 1, a_n = 2a_{n-2} - a_{n-1} + 1, n \geq 2.$

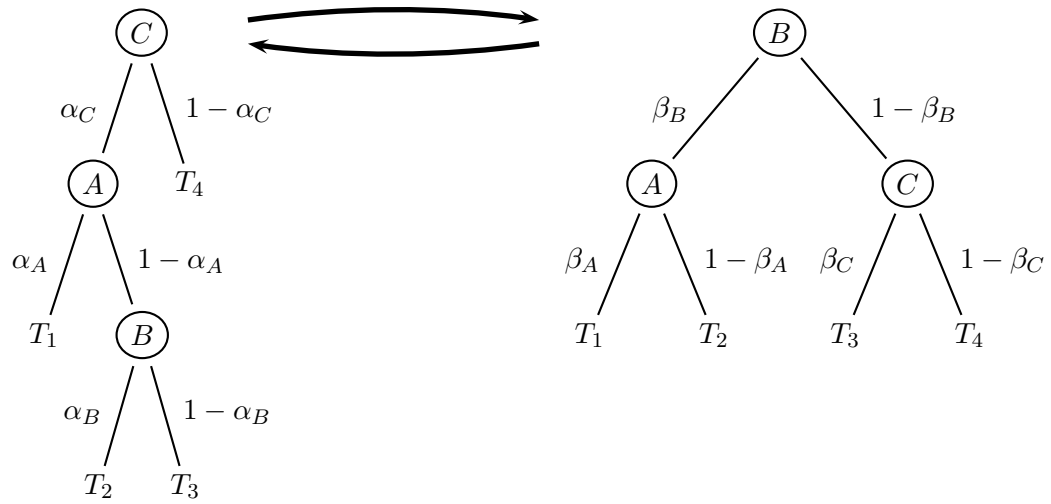
3. Rolling a Die.

You roll an ordinary six-sided die $n \geq 2$ times, recording the sequence of rolls.

- (a) Find a recurrence relation for the number of possible sequences in which there are no two consecutive sixes.
- (b) Solve the recurrence using annihilators; you need not solve the simultaneous equations from the initial conditions.
- (c) How does the probability that a sequence of n rolls of the die with have no two consecutive sixes grow as $n \rightarrow \infty$?

4. Double Rotations.

Consider the double rotation of a binary search tree shown below.



- (a) Explain why $\beta_B = \alpha_C \alpha_A + \alpha_C (1 - \alpha_A) \alpha_B$.
- (b) Express β_A and β_C in terms of α_A , α_B , and α_C .

5. Divide-and-Conquer

Having seen the power of recursion and divide-and-conquer, CS 330 student decided to write a program to compute x^n ,

(a) His first attempt was

```
function Power( $x, n$ )  
1: if  $n = 0$  then  
2:   return 1  
3: else if  $n$  is odd then  
4:   return  $x * \text{Power}(x, \lfloor n/2 \rfloor) * \text{Power}(x, \lfloor n/2 \rfloor)$   
5: else  
6:   return  $\text{Power}(x, \lfloor n/2 \rfloor) * \text{Power}(x, \lfloor n/2 \rfloor)$   
7: end if
```

Analyze the time required by this algorithm.

(b) His second attempt was

```
function Power( $x, n$ )  
1: if  $n = 0$  then  
2:   return 1  
3: else  
4:   integer  $t \leftarrow \text{Power}(x, \lfloor n/2 \rfloor)$   
5:   if  $n$  is odd then  
6:     return  $x * t * t$   
7:   else  
8:     return  $t * t$   
9:   end if  
10: end if
```

Analyze the time required by this algorithm.

5. Divide-and-Conquer, continued.