**Conditional Probability**

\[ P(E|F) = \frac{P(E \cap F)}{P(F)} \]

**Bayes' Law**

\[ P(E|F) = \frac{P(F|E) \cdot P(E)}{P(F)} \]
**Bayes' Theorem**

\[
\Pr(C|E) = \frac{\Pr(E|C) \Pr(C)}{\Pr(E)}
\]

\[
\Pr(\text{cervical cancer in a woman in her 40's}) = \frac{40}{10000}
\]

\[
\Pr(\text{positive mammogram among all women}) = \frac{1028}{10000}
\]

\[
\Pr(\text{positive mammogram among cancer patients}) = \frac{32}{40}
\]

\[
\Pr(\text{cancer in patient} | \text{positive mammogram}) = \frac{\Pr(\text{cancer in 40's}) \Pr(\text{positive M.} | \text{cancer in 40's})}{\Pr(\text{positive M.})} = \frac{40 \times 32}{10000 \times 40} \approx 0.03
\]

\[
\Pr(\text{gray}) = \Pr(\text{gray in 1}) \times \Pr(\text{Univ. 1}) + \Pr(\text{gray in 2}) \times \Pr(\text{Univ. 2}) + \Pr(\text{gray in 3}) \times \Pr(\text{Univ. 3}) = \frac{1}{3}
\]
\[ P(a \text{ wins } 1 \mid \text{gray}) = \frac{P(a \text{ wins } 1) \cdot P(\text{gray} \mid \text{a wins } 1)}{P(\text{gray})} \]
\[ = \frac{\frac{1}{3} \times 0}{\frac{1}{2}} = 0 \]
\[ P(a \text{ wins } 2 \mid \text{gray}) = \frac{P(a \text{ wins } 2) \cdot P(\text{gray} \mid \text{a wins } 2)}{P(\text{gray})} \]
\[ = \frac{\frac{1}{3} \times 1}{\frac{1}{2}} = \frac{2}{3} \]
\[ P(a \text{ wins } 3 \mid \text{gray}) = \frac{P(a \text{ wins } 3) \cdot P(\text{gray} \mid \text{a wins } 3)}{P(\text{gray})} \]
\[ = \frac{\frac{1}{3} \times \frac{1}{2}}{\frac{1}{2}} = \frac{1}{3} \]

\[ P(a \text{ wins } 1 \mid 2 \text{ gray bulbs}) = \frac{P(a \text{ wins } 1) \cdot P(2 \text{ gray bulbs} \mid a \text{ wins } 1)}{P(2 \text{ gray bulbs})} \]
\[ = \frac{\frac{1}{3} \times 0}{P(2 \text{ gray bulbs})} = 0 \]

\[ P(2 \text{ gray bulbs}) = P(a \text{ wins } 1) \cdot P(2 \text{ gray bulbs} \mid a \text{ wins } 1) \]
\[ + P(a \text{ wins } 2) \cdot P(2 \text{ gray bulbs} \mid a \text{ wins } 2) \]
\[ + P(a \text{ wins } 3) \cdot P(2 \text{ gray bulbs} \mid a \text{ wins } 3) \]
\[ = \frac{1}{3} \times 0 + \frac{1}{3} \times 1 + \frac{1}{3} \times \frac{1}{4} \]
\[ = \frac{5}{12} \]

\[ P(a \text{ wins } 2 \mid 2 \text{ gray bulbs}) = \frac{\frac{1}{3} \times 1}{\frac{5}{12}} = \frac{4}{5} \]
\[ P(a \text{ wins } 3 \mid 2 \text{ gray bulbs}) = \frac{\frac{1}{3} \times \frac{1}{4}}{\frac{5}{12}} = \frac{1}{3} \]
**Search**

**Balanced Trees**

**Lexicographic Tree**

**Dynamic**

```
          x
         /|
        /  \
       x   x
```

```
height \(O(\log n)\)
```

**Rotation**

```
alpha_A = P_{left \mid left \text{ or } A}
alpha_B = P_{left \mid right \text{ or } B}
beta_A = P_{left \mid \text{left or } C}
beta_B = P_{left \mid \text{right or } B}
```

Express \(beta\)'s in terms of \(alpha\)'s:

```
alpha_A = \frac{P_{<A}}{P_{<B} + \beta_A \cdot (1 - \beta_A)}
alpha_B = \frac{\beta_A}{\beta_A + (1 - \beta_A) \cdot \beta_B}
```

```
alpha = \beta_A + (1 - \beta_A) \cdot \beta_B
alpha_A = \frac{\beta_A}{\beta_A + (1 - \beta_A) \cdot \beta_B}
```