Math Induction

\[ 1 + 2 + 3 + \ldots + n = \frac{n(n+1)}{2} \]

truth for \( n \Rightarrow \) truth \( n + 1 \)

Harmonic numbers \( H_1, H_2, H_3, \ldots \)

\[ H_{n+1} = H_n + \frac{1}{n+1} \]

\[ H_1 = 1 \]

\[ H_n = 1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \ldots + \frac{1}{n} \]

\[ 2nH_n \]

\[ 1 + n \geq H_{2n} \geq 1 + \frac{n}{2} \]

\[ H_k \approx \log \text{ behavior growth} \]

\[ H_n = \Theta(\log k) \]

\[ \ln = \log_{10} \]

\[ \log = \log_2 \]

\[ \log_2 \text{ branch cut \( \pm \)} \]

\[ \log \]
\[
\begin{align*}
D_1 &= 1 \\
D_n &= D_{n-1} + \frac{1}{2n-1} \\
D_n &= \frac{\ln n}{e} \\
\text{Harmonic Numbers }
\end{align*}
\]
If there are $k$ people with black hats, then in the $k$th hour, everyone leaves.

BASE

\[
\begin{align*}
  h = 1 & \quad \text{OK} \\
  h = 2 & \quad \text{OK} \\
  h = 3 & \\
\end{align*}
\]

Statement $S_k \implies S_{k+1}$

\[S_i = \text{Everybody knows that at least } i \text{ people have black hats}\]

Claim: $S_i$ is true at $i$th hour

Proof by Induction
Euclid's Algorithm for G.C.D.

Define:
- \( u \geq v \)
- \( r_0 = u \)
- \( r_1 = v \)

Recurrence relation:
- \( r_{i+1} = r_{i-1} \mod r_i \)

Example:
- \( \text{gcd}(21, 13) = \text{gcd}(13, 8) \)
- \( 21 \mod 13 \rightarrow 8 \)

Worst case?

- \( r_{i-1} = q r_i + r_{i-2} \)
- \( q \geq 1 \) \( r_0 \geq r_1 \)
- \( r_{i-1} \geq r_i + r_{i-1} \)

Example steps:
- \( i = 1 \):
  - \( r_0 \geq r_1 - r_2 \implies r_2 + r_3 + r_2 = 2r_2 + r_3 \)
- \( i = 2 \):
  - \( r_1 \geq r_2 + r_3 \)
- \( i = 3 \):
  - \( r_2 \geq r_3 + r_4 \)
- \( i = 4 \):
  - \( r_3 \geq r_4 + r_5 \)
\[ r_0 \geq 5v_u + 3v_s \]

\[ \vdots \]

\[ \geq F_k r_k + F_{k-1} r_{k-1} \]

\[ \vdots \]

\[ 0, 1, 1, 2, 3, 5, 8, 13, 21, \ldots \]

FIBONACCI NUMBERS

\[ F_{k+1} = F_k + F_{k-1} \]

\[ F_0 = 0 \]

\[ F_1 = 1 \]

\[ 10^{n+1} > \]

\[ \Rightarrow a \approx 4.785n \]