

# Tautologies; Normal Forms

CS 350, Lecture 6, Mon Jan 30, 2012

*ver. Mon, Jan 30, 2012, 12:15 pm*

## Activity 6.1: Tautologies, Contradictions, and Contingencies

### A. Why?

- Tautologies describe properties that are always true and transformations that are always valid.

### B. Outcomes

At the end of today, you should:

- Be able to recognize tautologies, contradictions, and contingencies by their truth tables.

### C. Questions

1. Write a truth table to show that  $((X \text{ IMPL } Y) \text{ AND } (Y \text{ IMPL } Z)) \text{ IMPL } (X \text{ IMPL } Z)$  is a tautology.
2. Write a truth table to test one of DeMorgan's laws on three variables:  $\text{NOT}(X \text{ OR } Y \text{ OR } Z) = \text{NOT } X \text{ AND } \text{NOT } Y \text{ AND } \text{NOT } Z$ . Is this a tautology, contradiction, or contingency?
3. Write a truth table for  $((X \text{ IMPL } Y) \text{ IMPL } Z) \text{ IFF } (X \text{ IMPL } (Y \text{ IMPL } Z))$ ; is this a tautology, a contradiction, or a contingency?

## Activity 6.2: Disjunctive Normal Form

### A. Why?

- Disjunctive Normal Form (DNF) is a particular standard way to represent a logical expression.
- DNF corresponds to a sum of the rows of 1's for the truth table of an expression.

### B. Outcomes

At the end of today, you should:

- Be able to translate between an expression's disjunctive normal form (DNF) and its truth table.

**C. Questions**

1. Say an expression's truth table has  $n$  rows of 1's and its full DNF representation has  $m$  terms; what is the relationship between  $n$  and  $m$ ?
2. Translate  $\neg(X + YZ)$  into DNF. If the result is not in full DNF, then give also the equivalent full DNF representation. Finally, use the full DNF representation to write a truth table for the expression.
3. Translate  $(X + \bar{Y})(Y + Z)$  into DNF. (Feel free to drop terms that include a variable and its negation.) If the result is not in full DNF, then give also the equivalent full DNF representation. Finally, use the full DNF representation to write a truth table for the expression.
4. Write out an arbitrary truth table for an expression over (i.e., using the variables)  $X$ ,  $Y$ , and  $Z$ . Translate the truth table to the equivalent full DNF representation.

**D. Solutions**

1. [Problem not done yet.]
2.  $\neg(X + YZ) = \bar{X} \neg(YZ) = \bar{X}(\bar{Y} + \bar{Z}) = \bar{X}\bar{Y} + \bar{X}\bar{Z}$  is in DNF

For full DNF, we can expand

$$\begin{aligned} & \bar{X}\bar{Y} + \bar{X}\bar{Z} \\ &= \bar{X}\bar{Y}(Z + \bar{Z}) + \bar{X}(Y + \bar{Y})\bar{Z} \\ &= \bar{X}\bar{Y}Z + \bar{X}\bar{Y}\bar{Z} + \bar{X}Y\bar{Z} + \bar{X}\bar{Y}\bar{Z} \\ &= \bar{X}\bar{Y}Z + \bar{X}\bar{Y}\bar{Z} + \bar{X}Y\bar{Z} \text{ (removing repeated } \bar{X}\bar{Y}\bar{Z} \text{)} \end{aligned}$$

Here's the same work using the keyword notation for operators:

$$\begin{aligned} & \text{NOT}(X \text{ OR } Y \text{ AND } Z) \\ &= \text{NOT } X \text{ AND NOT } (Y \text{ AND } Z) \\ &= \text{NOT } X \text{ AND } (\text{NOT } Y \text{ OR NOT } Z) \\ &= \text{NOT } X \text{ AND NOT } Y \text{ OR NOT } X \text{ AND NOT } Z \quad (\text{plain DNF}) \\ &= \text{NOT } X \text{ AND NOT } Y \text{ AND } (Z \text{ OR NOT } Z) \\ & \quad \text{OR NOT } X \text{ AND } (Y \text{ OR NOT } Y) \text{ AND NOT } Z \\ &= \text{NOT } X \text{ AND NOT } Y \text{ AND } Z \\ & \quad \text{OR NOT } X \text{ AND NOT } Y \text{ AND NOT } Z \\ & \quad \text{OR NOT } X \text{ AND } Y \text{ AND NOT } Z \\ & \quad \text{OR NOT } X \text{ AND NOT } Y \text{ AND NOT } Z \end{aligned}$$

$$\begin{aligned}
 &= \text{NOT } X \text{ AND NOT } Y \text{ AND } Z \\
 &\quad \text{OR NOT } X \text{ AND NOT } Y \text{ AND NOT } Z \\
 &\quad \text{OR NOT } X \text{ AND } Y \text{ AND NOT } Z \quad (\text{full DNF})
 \end{aligned}$$

[Truth table part of question is yet to be done]

3. For plain DNF,

$$\begin{aligned}
 &(X + \overline{Y})(Y + Z) \\
 &= X(Y + Z) + \overline{Y}(Y + Z) \\
 &= XY + XZ + \overline{Y}Y + \overline{Y}Z \\
 &= XY + XZ + \overline{Y}Z
 \end{aligned}$$

For full DNF, we continue

$$\begin{aligned}
 &= (XYZ + XY\overline{Z}) + (XYZ + X\overline{Y}Z) + (X\overline{Y}Z + \overline{X}\overline{Y}Z) \\
 &= XYZ + XY\overline{Z} + X\overline{Y}Z + \overline{X}\overline{Y}Z
 \end{aligned}$$

[Truth table part of question is yet to be done]

4. [Problem not yet done]