

Notes: Logic Gates and Truth Tables

A. Why?

Basic logic gates are devices that perform simple functions on logical values. We can describe these functions using logical formulas or truth tables.

B. Outcomes

By the end of the class you should

- Know the behavior of some typical gates: NOT, AND, OR, NAND, NOR.
- Know how to write logical “propositional” formulas.
- Know what a truth table is and how to build one for a logical formula.
- Know how DeMorgan’s laws connect the behavior of AND, OR, and NOT.

C. Logic Gates

- NOT Gate
 - Use voltage inverter as NOT gate.
 - Voltage behavior can be viewed as logical function.
- Other Gates: AND, OR, NAND, NOR
 - $X \text{ AND } Y = \text{true iff (if and only if) both } X \text{ and } Y \text{ are true.}$
 - $X \text{ OR } Y = \text{true iff one or more of } X \text{ and } Y \text{ is true.}$
 - $X \text{ NAND } Y = \text{NOT } (X \text{ AND } Y) = \text{false iff both } X \text{ and } Y \text{ are true.}$
 - $X \text{ NOR } Y = \text{NOT } (X \text{ OR } Y) = \text{false iff one or more of } X \text{ and } Y \text{ are true.}$

D. Propositions and Truth Tables

- Propositions = propositional formulas = boolean formulas based on boolean variables.
- Two propositions are logically equivalent if they produce exactly the same truth values for all possible truth value parameters.
- A truth table describes the truth values of one or more formulas.
 - Columns correspond to logical values of variables or expressions.
 - One row for each possible combination of values for variables.
- DeMorgan’s Laws
 - Compare truth tables for $\text{NOT } (X \text{ AND } Y)$ and $(\text{NOT } X) \text{ OR } (\text{NOT } Y)$.
 - Also compare tables for $\text{NOT } (X \text{ OR } Y)$ and $(\text{NOT } X) \text{ AND } (\text{NOT } Y)$.
 - DeMorgan’s laws describe how $\text{NOT}(\text{some expression})$ can be translated into an expression without a top-level NOT.