1. (4 points) Exercise 15.3-4

2. (4 points) Write pseudocode to output the nodes in order for the optimal binary search tree given the \( r(i,j) \) array (the array that holds the root choices during the construction of the \( A \) array). The ordering should be top to bottom, left to right as shown below. Explain that this can be done in \( O(n) \) time.

Example BST with nodes numbered:

```
   dog,1
  /   \
cat,2   frog,3
 /    \
bird,4  cow,5
|
(empty),8 (empty),9
```

The output should look like

1. dog
2. cat
3. frog
4. bird
5. cow
6. (empty)
7. goat
8. (empty)
9. (empty)
10. cheetah
11. dingo
12. (empty)
13. (empty)
14. gecko
15. hamster

(Hint: Recursively walk through the \( r(i,j) \) array starting from \( r(1,n) \) and store the key values (or empty) in a global array.)

3. (4 points) Exercise 16.1-2

4. (4 points) Exercise 16.2-3

5. (4 points) In the art gallery guarding problem we are given a line \( L \) that represents a long hallway in an art gallery. We are also given a set \( X = \{X_0, X_1, ..., X_{n-1}\} \) of real numbers that specify the positions of paintings in this hallway. Suppose that a single guard can protect all the paintings within distance at most 1 of his or her position (on both sides). Design an algorithm for finding a placement of guards that uses the minimum number of guards to guard all the paintings with positions in \( X \). A guard can be placed at any position in the hallway. Prove the required properties of the problem and for your solution approach.

6. (5 points) A subsequence is a palindrome if it is the same when read left to right and right to left. A subsequence does not have to be contiguous. Describe a polynomial-time algorithm to find the longest subsequence which is a palindrome in a given string represented by an array \( A[1..n] \). For example, the string \( abcab \) has four palindromes of length 3: \( aba, aca, bcb, \) and \( bab \), but no palindrome of length 4. Prove the required properties of the problem and for your solution approach.