1 Minimum Spanning Trees Definitions

$G = (V, E)$ is an undirected graph whose edges have weight $w$. A spanning subgraph of $G$ is identified with the its set of edges.

Let $A$ be a set of edges $A \subseteq E$. We say that edge $e$ is safe for $A$ if the following property holds: if $A$ is contained in some minimum spanning tree, then $A \cup \{e\}$ is contained in some minimum spanning tree.

A **cut** $(S, \bar{S})$ of a graph is a partition of $V$ into two nonempty sets $S$ and $ar{S} = V \setminus S$.

We say that an edge $e$ crosses a cut $(S, \bar{S})$ if one of the endpoints of $e$ is in $S$ and the other endpoint is in $\bar{S}$.

We say that a cut respects a set of edges $A$ if no edge of $A$ crosses the cut.

An edge is a **light** edge crossing a cut if its weight is the minimum of any edge crossing the cut.

**Theorem 1.1 Blue Rule.** If the edge $e$ is light for some cut which respects the set of edges $A$, $e$ is safe for $A$. 