Solutions to Homework Assignment 9
CS 430 Introduction to Algorithms
Spring Semester, 2018

Solution:

1. To prove

\[ |OPT| \geq 2 \sum_{i=\lceil \frac{n}{2} \rceil + 1}^{n} l_i, \]

consider \( n \)’s parity:

- When \( n \) is even, it’s the same as the inequality (2) in the lecture. The proof has already been shown in the lecture notes.
- When \( n \) is odd, we denote \( n \) as \( n = 2k + 1 \). What we need to prove is

\[ |OPT| \geq 2 \sum_{i=k+2}^{2k+1} l_i \]

Following the same analysis in the lecture, we should have \( |OPT| \geq |T_{2k+1}| \geq \sum \min\{l_i, l_j\} \). Each \( l_i \) appears in this list at most twice. Still the edges were labeled in decreasing order, we can replace edges in the first \( k \) with members of the last \( k \) edges \( (l_{k+2}, l_{k+3}, \cdots, l_{2k+1}) \), this process yields:

\[ |OPT| \geq 2 \sum_{i=k+2}^{2k+1} l_i + l_{k+1} > 2 \sum_{i=k+2}^{2k+1} l_i \]

So we proved

\[ |OPT| \geq 2 \sum_{i=\lceil \frac{n}{2} \rceil + 1}^{n} l_i, \]

2. By the triangle inequality, the last edge is no longer than the sum of the lengths of the other edges; therefore that edge can contribute no more than 1/2 to the ratio \( \frac{|NN|}{|OPT|} \). The last edge is thus not the cause of the logarithmic ratio.