

Homework 4

Assigned: October 28

Due: November 9

Please respect the following guidelines for writing pseudocode:

1. C instructions are fine. But do not write object-oriented additions. Do not declare or use any class. Declare only procedures (if necessary) and explain in words what each procedure does, and what is the use of each parameter.
2. One instruction per line
3. Match the brackets with a horizontal line
4. Number your lines
5. Write down if your array is indexed $0 \dots n - 1$ or $1 \dots n$.
6. If you use results not covered in class, present full details of pseudocode, analysis, and proofs (to the point only results from the class are used).

Problem 1 What is the best way to multiply a chain of matrices with dimensions that are 10×5 , 5×2 , 2×20 , 20×12 , 12×4 , and 4×60 ? Show your work.

Problem 2 Suppose we are given a collection $A = a_1, a_2, \dots, a_n$ of n positive integers that add up to N . Design an $O(nN)$ -time algorithm for determining whether there is a subset $B \subseteq \{1, 2, \dots, n\}$, such that $\sum_{i \in B} a_i = \sum_{i \in \{1, 2, \dots, n\} \setminus B} a_i$.

Hint: Use dynamic programming.

Problem 3 Given an array A with n integers, a monotonically increasing subsequence is a sequence of (not necessarily consecutive) indexes $i_1 < i_2 < \dots < i_k$ such that $A[i_1] < A[i_2] < \dots < A[i_k]$; the length of subsequence is k . Given an $O(n^2)$ algorithm to find a longest monotonically increasing subsequence. Present pseudocode and analyze the running time.

Hint: Dynamic programming.

Problem 4 Prove that any sequence of $nm + 1$ integers has a monotonically increasing subsequence of length n or a monotonically decreasing subsequence of length m .