

Homework 5

Assigned: November 11

Due: November 23

Please respect the following guidelines for writing pseudocode:

1. C instructions are fine. But do not write object-oriented additions. Do not declare or use any class. Declare only procedures (if necessary) and explain in words what each procedure does, and what is the use of each parameter.
2. One instruction per line
3. Match the brackets with a horizontal line
4. Number your lines
5. Write down if your array is indexed $0 \dots n - 1$ or $1 \dots n$.
6. If you use results not covered in class, present full details of pseudocode, analysis, and proofs (to the point only results from the class are used).

Problem 1 Bob loves foreign languages and wants to plan his course schedule to take the following nine language courses: LA15, LA16, LA22, LA31, LA32, LA126, LA127, LA141, LA169. The course prerequisites are:

LA15: (none)	LA126: LA22, LA32
LA16: LA15	LA127: LA16
LA22: (none)	LA141: LA22, LA16
LA31: LA15	LA169: LA32
LA32: LA16, LA31	

Find a sequence of courses that allows Bob to satisfy all the prerequisites.

Problem 2 Let G be a graph whose vertices are the integers 1 through 8, and let the adjacent vertices of each vertex be given by the table below:

vertex	adjacent vertices
1	(2,3,4)
2	(1,3,4)
3	(1,2,4)
4	(1,2,3,6)
5	(6,7,8)
6	(4,5,7)
7	(5,6,8)
8	(5,7)

Assume that in a traversal of G , the adjacent vertices of a given vertex are returned in the same order as they are listed in the above table.

- a. Draw G .
- b. Order the vertices as they are visited in a DFS traversal starting at vertex 1.

c. Order the vertices as they are visited in a BFS traversal starting at vertex 1.

Problem 3 An independent set of an undirected graph $G=(V,E)$ is a subset I of V , such that no two vertices in I are adjacent. That is, if $u,v \in I$, then $(u,v) \notin E$. A *maximal independent set* M is an independent set such that, if we were to add any additional vertex to M , then it would not be independent any longer. Every graph has a maximal independent set. (Can you see this? This question is not part of the exercise, but it is worth thinking about.) Give an efficient algorithm that computes a maximal independent set for a graph G . What is this method's running time?

Present pseudocode and analyze the running time. $O(|V| + |E|)$ is achievable.

Problem 4 Give an algorithm that determines whether or not a given undirected graph $G = (V, E)$ contains a cycle. Your algorithm should run in in $O(|V|)$ time, independent of $|E|$.

Problem 5 Give an efficient algorithm to determine if an undirected graph is bipartite. Prove your algorithm is correct. The definition of a bipartite graph appears in the textbook, page 1083. $O(|V| + |E|)$ is achievable.