CS 430 Introduction to Algorithms **Spring Semester, 2021**

Homework 6

Assigned: April 21 *Due: May 5*

Problem 1 Present full pseudocode of a variant of Prim's algorithm that runs in time $O(|V|^2)$ for a graph $G = (V, E)$ given by an adjacency matrix A. Analyze the running time.

Note: do not use any procedure or data structures operation.

Problem 2 Give an example of a weighted directed graph $\vec{G}$ with negative-weight edges, but no negative-weight cycle, such that Dijkstra's algorithm incorrectly computes the shortest-path distances from some start vertex v. Use the algorithm version from the handout.

A four-vertex example is possible. Draw the graph, mention the start vertex, show the result of Dijkstra's algorithm, and point out for which vertex the result is incorrect.

(This was on a previous final exam)

Problem 3 A *complete digraph* has exactly one directed edge (also called *arc*) from every vertex u to every vertex v other than itself. Let G be a complete digraph with non-negative arc weights. Let the *capacity* of a path be the *minimum* arc weight along it, and let the *capacity* of a pair of nodes (u, v) be the *maximum* capacity of a path from u to v. Find a Dijkstra-like algorithm to find, for all $v \neq s$, the capacity of (s, v). (Node s is a fixed source.)

Present the pseudocode, analyze the running time, and prove correctness.

Problem 4 Assume that the weights on the edges of directed graph G are integers in the range from 1 to K. Give a new method for implementing EXTRACT-MIN(Q) and DECREASE-KEY($Q, v, d[v]$) so that Dijkstra's algorithm's running time becomes $O(|E| + K|V|)$.

Describe the data structure and present the pseudocode for the two operations used by Dijkstra's algorithm, together with correctness and running-time analysis.