## Minimum Spanning Trees

A tree is a connected graph with no cycles. A spanning tree is a subgraph of G which has the same set of vertices of G and is a tree.

A minimum spanning tree of a weighted graph G is the spanning tree of G whose edges sum to minimum weight.

There can be more than one minimum spanning tree in a graph  $\rightarrow$  consider a graph with identical weight edges.

The minimum spanning tree problem has a long history – the first algorithm dates back at least to 1926!.

Minimum spanning tree is always taught in algorithm courses since (1) it arises in many applications, (2) it is an important example where *greedy* algorithms always give the optimal answer, and (3) Clever data structures are necessary to make it work.

In greedy algorithms, we make the decision of what next to do by selecting the best local option from all available choices – without regard to the global structure.

## Applications of Minimum Spanning Trees

Minimum spanning trees are useful in constructing networks, by describing the way to connect a set of sites using the smallest total amount of wire. Much of the work on minimum spanning (and related Steiner) trees has been conducted by the phone company.

Minimum spanning trees provide a reasonable way for *clustering* points in space into natural groups.

When the cities are points in the Euclidian plane, the minimum spanning tree provides a good heuristic for travelling salesman problems. The optimum traveling salesman tour is at most twice the length of the minimum spanning tree.



The Option Traveling System tour is at most twice the length of the minimum spanning tree.

Note: There can be more than one minimum spanning tree considered as a group with identical weight edges.