

CS 430 - Fifth Quiz

Please work on separate paper, and write only the final solution here. Also, check your work as you are not able to run the code, so it better be right. Use only pens, blank paper, and the notes distributed in class. You have 40 minutes. Good luck!

Problem 1

Write pseudocode code for the problem below. If you decide to use procedures/functions/methods from the class (such as, say, DELETE-MAX(S) for S a max-heap), explain clearly what they do and in what running time. Explain the data structures, if any.

Suppose that you are given an $n \times n$ checkerboard and a checker. You must move the checker from the bottom edge of the board to the top edge of the board according to the following rule. At each step you may move the checker to one of three squares:

1. the square immediately above,
2. the square that is one up and one to the left (but only if the checker is not already in the leftmost column),
3. the square that is one up and one to the right (but only if the checker is not already in the rightmost column).

Each time you move from square x to square y , you receive $p(x, y)$ dollars. You are given $p(x, y)$ for all pairs (x, y) for which a move from x to y is legal. Do **not** assume that $p(x, y)$ is positive.

Give an efficient algorithm that figures out the set of moves that will move the checker from somewhere along the bottom edge to somewhere along the top edge while gathering as many dollars as possible. Your algorithm is free to pick any square along the bottom edge as a starting point and any square along the top edge as a destination in order to maximize the number of dollars gathered along the way. For full marks prove an $O(n^2)$ running time for your algorithm.

Argue that the result of the algorithm is correct, and that the running time you derive is correct.

Hint (and this helps with arguing correctness): write down what the quantities computed are, and then the recurrence relations.