**Min Spanning Trees**

$G = (V, E)$

$ST = (V, T)$

$T \subseteq E$

No cycle

$O(E + |V| \log |V|)$

Steiner Points

Min Steiner Tree

No known polynomial time algorithm!

None is believed to exist!

Prim's Algorithm

- Find shortest edge that doesn't add to $S$. $T$
- Add to $S$. $T$
- Delete edge from graph

until $S$. $T$ is finished

1) Analyze
2) Correct?

Fib heap

- $\text{extract-min} \in \Theta(\log n)$
- $\text{merge} \in O(1)$

Multiple vertices

Distance is closest point w.r.t.
Prim's

$O(|V|^2)$

draw

Extract min

Current? M.S.T.? 

Suppose not - look at first mistake

M.S.T.
Kruskal's Algorithm:

- Edges, sorted by length \( O(\log n) \)

Odd, unorder, edges that don't cause a cycle w/ previously added edges.

```
union/find
MAKESET for each vertex
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```
\begin{align*}
\forall u \in V &: \left[ \text{find}(u) = \text{parent} \right] = \text{parent} \\
\forall e \in E &: \left[ \text{find}(u) = \text{parent} \right] \\
\end{align*}
```

- A.F. : \( \alpha(n) \)

- \( 0 \left( |E| \alpha(n) \right) \)
Shortest Path

**BFS** Single source / multi destination

\[ O(1V_1 + 1E_1) \]

**Dijkstra's**

Add length to edge

\[ 1O_1 + O(1V_1 1V_1) \]

Priority Queue

\[ O(1V_1) \]

\[ \text{edge length positive} \]

\[ \text{negative cycle} \]

\[ A \rightarrow \rightarrow \rightarrow \rightarrow \rightarrow t \]