AMORTIZED ANALYSIS

\[ \text{Total cost} = \sum_{i=1}^{n} \text{cost}(\theta_i) \]

1) Aggregate
2) Accounts method
3) Potential function method $\Phi$ — rule for managing heap accounts

Dynamic Tables (Hash Table)

Full table — double the size

- Insert $x$ — use $x$ and table size
to compute where to put $x$

Full table — double table size AND reinsert all items into bigger table

Aggregate

<table>
<thead>
<tr>
<th>Insertion</th>
<th>Cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>$1$</td>
<td>$1$</td>
</tr>
<tr>
<td>$2$</td>
<td>$1$</td>
</tr>
<tr>
<td>$3$</td>
<td>$1$</td>
</tr>
<tr>
<td>$4$</td>
<td>$1$</td>
</tr>
<tr>
<td>$5$</td>
<td>$1$</td>
</tr>
<tr>
<td>$6$</td>
<td>$1$</td>
</tr>
<tr>
<td>$7$</td>
<td>$1$</td>
</tr>
<tr>
<td>$8$</td>
<td>$1$</td>
</tr>
<tr>
<td>...</td>
<td>...</td>
</tr>
</tbody>
</table>

$2^{n+1}$

$1 + 2 + 4 + 8 + \ldots + 2^n = 2^{n+1}$

$= 2n$

$3n$
Accurate Method

Simple insertion $\rightarrow$ $3

$1 \text{ labor}

$1 \text{ around security

$1 \text{ IRA - attach $5 to item

$1 \text{ IRA

Doubling insertion $\rightarrow$ $3

$1 \text{ labor

$1 \text{ SSA

$1 \text{ IRA

P/I by induction

Potential Function

\[ \phi(t) \] such as:

- size of table
- number of items in the table

Type $\phi(t) = \alpha t + \beta n$

<table>
<thead>
<tr>
<th>Initial Table</th>
<th>$a = n$</th>
<th>$\alpha t + \beta n = n$</th>
</tr>
</thead>
<tbody>
<tr>
<td>First after doubling</td>
<td>$a = 2n$</td>
<td>$\alpha 2t + \beta n = 2n$</td>
</tr>
</tbody>
</table>

\[ \phi(t) = 2n - 1 \]

\[ \begin{align*}
\alpha + \beta &= 1 \\
2\alpha + \beta &= 0 \\
\alpha &= -1 \\
\beta &= 2
\end{align*} \]
\[ \Phi(\text{table}) = 2n - 2 \]

\[ \text{AMORT COST} = \frac{\text{ACTUAL COST}}{n} + \left( \frac{\Phi(n)}{n} \right) - \frac{\Phi(n)}{n^2} \]

Simple insertion:

\[ \approx 1 + (2^{(n+1)} - 2) - (2^n - 2) \]

\[ = 2 \]

Dataless insertion:

\[ \approx \frac{2^{(n+1)}}{n} + \frac{(2^n - 2)}{n^2} - \frac{(2^n - 2)}{n^2} \]

\[ = 2 \]

Deletion:

- Contract when too sparse
- Expand when full

Deletable when full

Cut in half when full (half empty)

\[ \frac{1}{4} \]

\[ \frac{1}{3} \]

\[ \frac{1}{2} \]

\[ \cdots \]

\[ \Phi(\text{table}) = \alpha \cdot n + \beta \cdot n \]