1. Construct DFAs that recognize the following languages over the alphabet \{a, b\}:

(a) \{w \mid w \text{ starts with } a \text{ and has an even length, or starts with } b \text{ and has an odd length}\}

(b) \{w \mid w \text{ contains as substring either } ababb \text{ or } bba \}

(c) \{w \mid w \text{ begins with a } b \text{ and ends with an } a\}.

(d) \{w \mid w \text{ does not contain the substring } abb\}

(e) \{w \mid |w| \leq 4\}

Draw the state diagram for all; add a formal description of the DFA for (b), and add a proof that your DFA recognizes exactly the specified language for (c). To do this, you will need to prove that (1) your DFA accepts all strings in the language and (2) any string accepted is in the language.

2. For the NFA in Figure 1, convert it to a DFA using the method of Theorem 20 from the notes.

3. For each of the following give two strings that are members of the language, and two strings that are not members of the language. Assume the alphabet \(\Sigma = \{0, 1\}\).

1. \(\Sigma^* 10 \Sigma^* 0 \Sigma^*\).

2. \((0 \cup 01 \cup 00)^*\)

3. \((011)^*\).

4. Give regular expressions generating the languages of Exercise 1. For (d), prove that the language of your expression is equal to the language as described in (d).
5. Let $A$ be a language. Define $A^R = \{ y \mid \exists w \in A \ y = w^R \}$. Prove that if $A$ is regular, $A^R$ is regular.

6. Prove that the following languages over $\{0, 1\}$ are not regular:
   1. $\{0^n10^n \mid n \geq 0\}$
   2. $\{w \mid \text{the number of 1's in } w \text{ is the square of the number of 0's in } w\}$
   3. $\{yvy \mid y, v \in \{0, 1\}^*\}$

7. Submit your signed academic integrity pledge with this homework.