1. Let $\Sigma$ be an alphabet of size $s$ ($s \geq 1$).

(a) How many strings of length $n$ can be formed with symbols from $\Sigma$?

(b) How many strings of length at most $n$ can be formed with symbols from $\Sigma$?

Justify your answers.

2. Construct DFAs that recognize the following languages over the alphabet $\{a, b\}$:

(a) $\{w \mid w$ contains exactly two $b$'s, or an even number of $a$'s$\}$

(b) $\{w \mid w$ contains as substring either $ababb$ or $bbb \}$.

(c) $\{w \mid w$ is any string except $abba$ or $aba\}$

Draw the state diagram for all; add a formal description of the DFA for (b), and add a proof that your DFA recognizes exactly the specified language for (c). To do this, you will need to prove that (1) your DFA accepts all strings in the language and (2) any string accepted is in the language.

3. Problem 1.16 (a) page 86 of the textbook Sipser - second edition (convert NFA to DFA). Please see the scan on the web page.

4. For each of the following give two strings that are members of the language, and two strings that are not members of the language. Assume the alphabet $\Sigma = \{0, 1\}$.

1. $\Sigma^*00\Sigma^*1\Sigma^*$.

2. $(1 \cup 10 \cup 00)^*$

3. $(111)^*$

5. Give regular expressions generating the languages of Exercise 2. For (a), prove (by induction) that the language of your expression is equal to the language as described in (a).

6. Problem 1.21(a) page 86 of the textbook Sipser - second edition (convert DFA to regular expression). Please see the scan on the web page.

7. Construct state diagrams of NFA's accepting languages specified by the following regular expressions:

1. $(21)^* \cup 2^*$
2. \((213)^*2^*\)
3. \((12 \cup 11)^*(21 \cup 22)^*\)

8. Prove that the following languages over \(\{a, b, c\}\) are not regular:

1. \(\{c^n b^n a^n \mid n \geq 0\}\)
2. \(\{w \mid w = w^R\}\)
3. \(\{w \mid w \neq w^R\}\)
4. \(\{a^n b^m \mid n \neq m\}\)