

## Homework 1

Assigned: Jan 25

Due: Feb 8

1. Construct DFAs that recognize the following languages over the alphabet  $\{a, b\}$ :
  1.  $\{w \mid w \text{ contains at least two } a\text{s and at most one } b\}$
  2.  $\{w \mid w \text{ does not contain the substring } bba\}$
  3.  $\{w \mid w \text{ contains at least three } b\text{s}\}$
  4.  $D = \{w \mid w \text{ has an equal number of occurrences of } ab \text{ and } ba \text{ as substrings}\}$ . Thus  $bab \in D$  because  $bab$  contains a single  $ab$  and a single  $ba$ , but  $baba \notin D$  because  $baba$  contains two  $ba$ s and one  $ab$ .

Draw the state diagram for all; add a formal description of the DFA for **2**).

2. For the NFA in Figure 1, convert it to a DFA using the method of Theorem 21 from the notes.

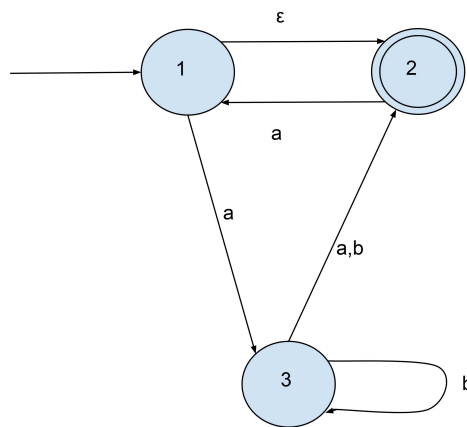


Figure 1: NFA for Problem 2.

3. Give regular expressions generating the languages of Exercise 1.
4. For each of the following give two strings that are members of the language, and two strings that are not members of the language. Assume the alphabet  $\Sigma = \{0, 1\}$ .
  1.  $0^*1^*$ .

2.  $1(01)^*0$ .
3.  $\Sigma^*0\Sigma^*1\Sigma^*0\Sigma^*$ .
4.  $(0110^* \cup 0 \cup 10)^*$ .
5.  $1^* \cup 0^*$ .

5. Obtain the regular expression that describes the same language as the automaton in Figure 2, using the method of Lemma 30 from the notes. Show your intermediate steps.

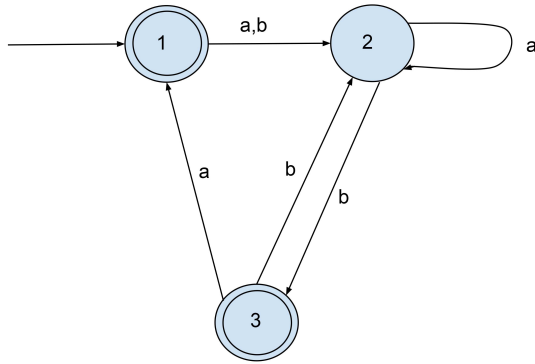


Figure 2: Finite Automaton for Problem 5.

6. Prove that the following languages over  $\{0, 1\}$  are not regular:

1.  $\{0^{p-1} \mid p \text{ is a prime}\}$
2.  $\{0^n 1^m 0^n \mid n, m \geq 0\}$
3.  $\{0^n 1^m \mid n \neq m\}$