

Homework 2

Assigned: Feb. 8

Due: Feb. 22

1. For each of the following (the alphabet $\Sigma = \{0, 1\}$), give context-free grammars that generate the language:

1. $\{w \mid w \text{ contains more 1s than 0s}\}$
2. $\{w \mid \text{the length of } w \text{ is odd and its middle symbol is a 0}\}$
3. $\{w \mid w = \varepsilon \text{ or } w \text{ ends and starts with the same symbol}\}$

2. Present state diagrams of push-down automata for the three languages above.

3. Show that the class of context-free languages is closed under concatenation. That is, show that if L_1 and L_2 are context-free languages, then $L_1 \circ L_2$ is context-free. This was a midterm problem some years back.

Show that the class of context-free languages is closed under union. That is, show that if L_1 and L_2 are context-free languages, then $L_1 \cup L_2$ is context-free. This was a midterm problem some years back.

Show that the class of context-free languages is closed under the star operation. That is, show that if L_1 is a context-free languages, then L_1^* is context-free.

4. Use the previous exercise to give another proof that every regular language is context free, by giving for every regular expression a CFG generating the same language.

5. Use the pumping lemma to prove that the following languages are not context-free. The alphabet is $\{a, b, \#\}$. Both were midterm problems.

1. $\{w\#x \mid w \text{ is a substring of } x, \text{ where } x \in \{a, b\}^*\}$
2. $\{a^i b^j \mid j \text{ divides } i\}$