Spring Semester, 2018

## Homework 3

Assigned: March 8

Due: March 29

1. This exercise concerns TM  $M_1$  whose description and state diagram appear in the scanned example (from Sipser first edition), which is design to recognize the language  $L = \{y \in \{0, 1, \sharp, \}^* \mid \exists w \in \{0, 1\}^* \ y = w \sharp w\}$ . In each of the parts, give the sequence of configurations that  $M_1$  enters when started on indicated input string.

- $1. \ 00.$
- 2. 0<sup>#</sup>0.
- 3. 0<sup>#</sup>#0.
- 4. 00#01.
- 5. 01#01.

**2.** Give state diagrams (pictures) for a Turing Machine that decides the following language over the alphabet  $\{0.1\}$ :  $\{w \mid w \text{ contains twice as many 1s as 0s}\}$ .

**3.** If  $M_1$  and  $M_2$  are two (not necessarily halting) Turing machines, then there exists a Turing machine M such that  $L(M) = L(M_1) \cup L(M_2)$ . Prove the set equality, and be aware of infinite loops. Feel free to use multitape but do not use nondeterministic Turing Machines.

4. A Turing machine with stay put instead of left is just like the normal TM except the transition function, which is:

$$\delta: Q \times \Gamma \to Q \times \Gamma \times \{R, S\}.$$

The machine can only move its head right, or let it stay in the same position. Show that this type of Turing machine is *not* equivalent to the usual version; that is exibit a language one type can recognize (give a state diagram here) and the other cannot (argue that the other type cannot recognize that language).