CS 530 Theory of Computation	**Spring Semester, 2022**

Homework 4

Assigned: March 29 | *Due: April 12*

1. An infinite set is called countable if its elements can be placed in a one-to-one and onto correspondence with natural numbers. The set of natural numbers, $\{0, 1, 2, 3, \ldots\}$, is denoted in the following by $\mathcal{N}$. Which of the following sets are countable? Prove your answers.

(a) The set of all subsets of size 2 of $\mathcal{N}$.

(b) The set of all finite subsets of $\mathcal{N}$.

(c) The set of all sets $\mathcal{N}_{a,b} = \{a \times i + b \mid i \in \mathcal{N}\}$, for $a, b \in \mathcal{N}$.

(d) The set of all subsets of $\mathcal{N}$.

2. Give a reduction (preferably a **mapping** reduction) showing that the language $OVERBOARD_{TM} = \{< M > \mid M$ is a TM and for no input does M attempt to move its head left from the leftmost tape cell$\}$ is not decidable.

3. Show that for any language A, A is decidable if and only if $A \leq_m \{1^n 0^{2n} \mid n \geq 0\}$. Show that for any language A, A is Turing-recognizable if and only if $A \leq_m A_{TM}$.