1. Let \( I = \{ \langle P \rangle \mid P \text{ is a pushdown automaton and the language } L(P) \text{ is infinite} \} \). Show that \( I \) is decidable.

2. Let \( L_1 \) and \( L_2 \) be two languages such that there exist no string \( w \) that belongs to both \( L_1 \) and \( L_2 \). Prove that, if \( L_1 \) and \( L_2 \) are both co-Turing-recognizable, there exists a decidable language \( A \) such that \( L_1 \subseteq A \) and \( L_2 \subseteq \overline{A} \).

3. Let \( C \) be a decidable language such that \( C \) consists of the descriptions of deciders: \( C = \{ \langle D_1 \rangle, \langle D_2 \rangle, \langle D_3 \rangle \ldots \} \). Show that, there exists a decidable language \( L \) that is not decided by any \( D_i \) such that \( \langle D_i \rangle \in C \).

4. Consider the problem of determining whether a Turing machine has a state that is never entered on any input. We denote this problem \( DWSN \). Formulate \( DWSN \) as a language and prove that this language is undecidable.