1. An infinite set is called countable if its elements can be placed in a one-to-one and onto correspondence with natural numbers. The set of natural numbers, \( \{0, 1, 2, 3, \ldots\} \), is denoted in the following by \( \mathcal{N} \). Which of the following sets are countable? Prove your answers.

(a) The set of all subsets of size 2 of \( \mathcal{N} \).

(b) The set of all finite subsets of \( \mathcal{N} \).

(c) The set of all sets \( \mathcal{N}_{a,b} = \{a \times i + b \mid i \in \mathcal{N}\} \), for \( a, b \in \mathcal{N} \).

(d) The set of all subsets of \( \mathcal{N} \).

2. Let \( C \) be a language over an alphabet \( \Sigma \) and assume the symbol ”;” does not belong to \( \Sigma \). Prove that \( C \) is Turing-recognizable if and only if a decidable language \( D \) exists such that \( C = \{x \mid \exists y (x; y \in D)\} \). Here \( x; y \) denotes the concatenation of the word \( x \), the symbol \( ; \), and the word \( y \).

**Hint:** For one direction of the proof, you get to choose what \( y \) would represent. Have \( y \) represent the number of steps a Turing Machine takes to finish.

3. Show that for any language \( A \), if \( A \) is Turing-recognizable, and \( A \leq_m \bar{A} \), then \( A \) is decidable.

4. Give a reduction (preferably a mapping reduction) showing that the language \( OVERBOARD_{TM} = \{< M > \mid M \text{ is a TM and for no input } M \text{ moves its head left when the head is on the leftmost tape cell}\} \) is not decidable.