1. Let $S = \{ \langle M \rangle \mid M \text{ is a TM that accepts } w^R \text{ whenever it accepts } w \}$. Prove that $S$ is an undecidable language.

2. Prove that the language

$$HALT - ON - BLANK - TAPE_{TM} = \{ \langle M \rangle \mid M \text{ is a TM and } M \text{ halts when started on the blank tape} \}$$

is not decidable. (Hint: show that $HALT_{TM} = \{ \langle M \rangle, w \mid M \text{ is a TM and } M \text{ halts on } w \} \text{ reduces to } HALT - ON - BLANK - TAPE_{TM}$. Don’t get confused by the fact that $HALT - ON - BLANK - TAPE_{TM}$ reduces trivially to $HALT_{TM}$, we need the other direction. The idea is to construct for each pair $\langle M \rangle, w$ a Turing machine that halts on the blank tape if and only if $M$ halts on $w$. )

3. Let $A, B$ be two languages over $\{0, 1\}^*$. Assume $A \leq_m B$, and $B$ is a regular language. Prove or disprove: does it follow that $A$ is regular?

4. Show that the class of NP languages is closed under the operation of concatenation. That is, show that if $A$ and $B$ are in NP, then the language $L = \{ xy \mid x \in A \text{ and } y \in B \}$ is also in NP.
   
   Also, show that NP is closed under the union and the star operations.