CS 530 Theory of Computation **Spring Semester, 2022**

Homework 5

Assigned: April 12 *Due: April 26*

Special late policy: all homeworks must be submitted online by 11:59PM April 30 (10% penalty if not submitted by April 26). Students taking the exam on the main campus must turn in the hard copy before the exam.

1. Show that P is closed under union, concatenation, complement, and the star operation.
Hint (for star): Use dynammic programming.

2. Show that, if $P = NP$, then every language $A \in P$ except $A = \emptyset$ and $A = \Sigma^*$ is NP-Complete.

3. EXACT3COVER (X3C) is the following problem: given a positive integer m, and a sequence of subsets $S_1, S_2, ..., S_r \subseteq U = \{1, 2, ..., m\}$, such that $|S_i| = 3$ for all i, determine if there is a subset $T \subseteq \{1, 2, \ldots, r\}$ such that $\cup_{j \in T} S_j = U$ and $\forall i, j \in T$ with $i \neq j$ we have $S_i \cap S_j = \emptyset$ (Such a set T of $m/3$ disjoint sets whose union is U is called an *exact cover* of U.)

Formulate X3C as a language. You can assume this X3C language is NP-hard and that all the theorems listed in the notes are true. Prove that X4C (formulate it as a language in a similar manner), described below, is NP-hard:

EXACTCOVERBY4SETS (X4C) is the following problem: given a positive integer m, and a sequence of subsets $S_1, S_2, ..., S_r \subseteq U = \{1, 2, ..., m\}$, such that $|S_i| = 4$ for all i, determine if there is a subset $T \subseteq \{1, 2, \ldots, r\}$ such that $\cup_{j \in T} S_j = U$ and $\forall i, j \in T$ with $i \neq j$ we have $S_i \cap S_j = \emptyset$