The penalty for being late is changed for this homework: it must be turned in before the final exam (May 2), with a 10% penalty. No further extensions are possible.

1. Show that, if $P = NP$, a polynomial time algorithm exists that, given a satisfiable boolean formula $\Phi$, actually produces a satisfying assignment for $\Phi$. (Note: $NP$ is a class of languages, and producing an assignment for a formula is a function. Thus simply saying that "because $SAT$ is in $NP$, we are done" is not enough.)

2. Knapsack is the following problem: Given integers $K$ and $B$, and items $\{1, 2, \ldots, n\}$ each with integer size $s_i$ and integer profit $p_i$, find a subset $S \subseteq \{1, 2, \ldots, n\}$ such that $\sum_{i \in S} s_i \leq B$ and $\sum_{i \in S} p_i \geq K$. The common optimization problem is: we attempt to maximize $K$, the total profit, with a bound $B$ on the total size of items one can pick.

   Formulate Knapsack as a language and prove the language is NP-Complete. The reduction must come from a problem claimed NP-Complete in the notes (use all the theorems from the notes even if we did not prove it yet in class).

3. Given that the Clique problem, i.e. the problem of finding a clique of size $k$, is NP-Hard (by a reduction from 3-SAT) show that the Independent set (IS), the problem of finding an independent set of size $l$, is NP-Hard. A set of vertices is independent if there is no edge between any two vertices in the set. For this problem, do not use Theorem 16 of the notes.

   Show that there exists a polynomial time reduction from IS to 3-SAT. (Hint: use theorems from the handout and do not give an explicit reduction here).