1. Show that $P$ is closed under union, concatenation, complement, and the star operation.
   
   Hint (for star): Use dynamic programming.

2. Show that, if $P = NP$, then every language $A \in P$ except $A = \emptyset$ and $A = \Sigma^*$ is NP-Complete.

3. EXACT3COVER (X3C) is the following problem: given a positive integer $m$, and a sequence of subsets $S_1, S_2, \ldots, S_r \subseteq U = \{1, 2, \ldots, m\}$, such that $|S_i| = 3$ for all $i$, determine if there is a subset $T \subseteq \{1, 2, \ldots, r\}$ such that $\cup_{j \in T} S_j = U$ and $\forall i, j \in T$ with $i \neq j$ we have $S_i \cap S_j = \emptyset$ (Such a set $T$ of $m/3$ disjoint sets whose union is $U$ is called an exact cover of $U$.)

   Formulate X3C as a language. You can assume this X3C language is NP-hard and that all the theorems listed in the notes are true. Prove that X4C (formulate it as a language in a similar manner), described below, is NP-hard:

   EXACTCOVERBY4SETS (X4C) is the following problem: given a positive integer $m$, and a sequence of subsets $S_1, S_2, \ldots, S_r \subseteq U = \{1, 2, \ldots, m\}$, such that $|S_i| = 4$ for all $i$, determine if there is a subset $T \subseteq \{1, 2, \ldots, r\}$ such that $\cup_{j \in T} S_j = U$ and $\forall i, j \in T$ with $i \neq j$ we have $S_i \cap S_j = \emptyset$

4. Consider the following problem, called DOMINATING-SET: Given a graph $G = (V, E)$, and an integer $k$, is there a set of vertices $A \subseteq V$ such that $|A| = k$ and every vertex of $V$ is either in $A$ or has a neighbour in $A$.

   Formulate DOMINATING-SET as a language and prove that this language is NP-hard. **Hints:** If you plan to use 3SAT, use three vertices for each variable and one vertex for each clause. If you plan to use VERTEX-COVER, add $m + 2$ vertices, where $m$ is the number of edges in the original instance.

   Of course, there are correct solutions which ignore the hints.