CS530: Introduction to Computational Complexity	Xiang-Yang Li
Homework 1	
Due: Sept 16, 2008	*YanWei Wu*

Problem 1: **Designing DFAs and NFAs.** For each of the following, draw a state diagram for a DFA or NFA (as required) that recognizes the specified language. In all cases the alphabet is $\{0, 1\}$.

(a) Sipser exercise 1.6, part c. $L_1 = \{w|w$ contains the substring $0101\}$. Provide a DFA recognizing L_1. Notice that you are required to write DFA, not NFA here.

(b) Sipser exercise 1.6, part j. $L_2 = \{w|w$ contains at least two $0s$ and at most one $1\}$. Provide a DFA recognizing L_2. Notice that you are required to write DFA, not NFA here.

(c) Sipser exercise 1.7, part c. $L_3 = \{w|w$ contains an even number of $0s$, or exactly two $1s\}$. Provide an NFA with at most six states that recognizes L_3. Notice that you can use NFA here.

Problem 2: **Proving an FA recognizes a language.** For one of the automata you designed in problem 1, prove that the machine recognizes *exactly* the specified language. To do this, you will need to prove that your automaton (1) accepts all strings in the language and (2) does not accept any string not in the language.

Problem 3: Given two languages A, B that are regular, show that the following languages are regular

1. $A \dagger B = \{a_1b_1a_2b_2a_3b_3 \cdots a_nb_n \mid n \geq 1, a_1a_2a_3 \cdots a_n \in A,$ and $b_1b_2b_3 \cdots b_n \in B\}$
2. $A \# B = \{a_1b_1a_2b_2a_3b_3 \cdots a_nb_n \mid n \geq 1, a_1a_2a_3 \cdots a_n \in A,$ or $b_1b_2b_3 \cdots b_n \in B\}$
3. $A \ddagger B = \{a_1a_2b_1a_3a_4b_2a_5a_6b_3 \cdots a_{2n-1}a_{2n}b_n \mid n \geq 1, a_1a_2a_3 \cdots a_{2n-1}a_{2n} \in A,$ and $b_1b_2b_3 \cdots b_n \in B\}$

Problem 4: Assume that the alphabet is $\Sigma = \{a, b, c\}$. Let $N(w, a)$ denote the number of times that character a appeared in string w. Write regular expressions for each of the following languages

1. $L_1 = \{w \mid N(w, a) \textit{ is odd}\}$.
2. $L_2 = \{w \mid N(w, b) \textit{ is even}\}$,
3. $L_3 = \{w \mid N(w, a) \textit{ is odd, and } N(w, b) \textit{ is even}\}$,

Problem 5: Describe the following language by nature language

1. 10*1
2. (0|1)*011(0|1)*

Problem 6: Prove that the following languages are non-regular:

1. $L_1 = \{0^i1^j0^k : j = \max\{i, k\}\}$
2. $L_2 = \{w|w = x\bar{x}, x \in \{0, 1\}^*, \bar{x}$ is the complement of $x\}$
3. Palindrome $L_p = \{w \mid w$ is a palindrome$\}$. Assume that the alphabet is $\Sigma = \{0, 1\}$. A palindrome is a binary string which is equivalent to its reversal. For example, 1, 0110 and 010111010 are palindromes, but 011 is not.