1.5em

CS530: Introduction to Computational Complexity Xiang-Yang Li

Homework 2

Due: Oct 2nd, 2008 *YanWei Wu*

Problem 1: Sipser problem 1.45 (page 90). Let $A/B = \{w \mid wx \in A \text{ for some } x \in B\}$. Show that if A is regular and $B = \Sigma^*$ (B is a language containing all possible strings), then A/B is regular.

Problem 2: Sipser exercise 2.4 and 2.6 (page 128). Given contex free grammmars that generate the following languages. In all parts, the alphabet Σ is $\{0, 1\}$.

1. $\{w \mid w \text{ starts and ends with the same symbol}\}$
2. $\{w \mid \text{ the length of } w \text{ is odd}\}$
3. $\{w \mid w = w^R\}$. Here w^R is the reverse of w.
4. The complement of the language $\{0^n 1^n \mid n \geq 0\}$, i.e., $\{w \mid w \text{ not in format } 0^n 1^n\}$.
5. $\{w \mid w \text{ has equal number of 0s and 1s}\}$.
6. $\{0^m 1^n \mid m \neq n, \text{ and } 2m \neq n\}$

Problem 3: Sipser problem 2.20 (page 130). Let $A/B = \{w \mid wx \in A \text{ for some } x \in B\}$. Show that if A is context-free and B is *regular* language, then A/B is context-free.

Problem 4: Given pushdown automatas that generate the following languages. You have to explicitly explain the meaings of the states used in your automata.

1. $\{w \mid w \text{ has equal number of 0s and 1s}\}$. Here the alphabet $\Sigma = \{0, 1\}$.
2. $\{a^i b^j c^k \mid i = j, \text{ or } j = k, i, j, k \geq 0\}$. Here the alphabet $\Sigma = \{a, b, c\}$.

Problem 5: Assume that you are given the DFA automata $M_A = (Q_A, \Sigma_A, q_a, F_A)$ for a regular language A and the DFA automata $M_B = (Q_B, \Sigma_B, q_b, F_B)$ for a regular language B. Construct a pushdown automata for the following language using M_A and M_B. $A \diamond B = \{xy \mid x \in A \text{ and } y \in B \text{ and } |x| = |y|\}$. Here $|x|$ is the length of the string x.

Problem 6: Prove that the following languages are *not context-free*.

1. $L_1 = \{w \mid N(w, a) = N(w, b) = N(w, c)\}$. Here the alphabet $\Sigma = \{a, b, c\}$ and $N(w, a)$ denotes the number of a's in the string w.
2. $L_2 = \{w \mid N(w, a) \text{ is a prime number}\}$. Here the alphabet $\Sigma = \{a, b\}$.