# Homework 4

*Due: Nov 13th, 2008* *YanWei Wu*

**Problem 1**: Show that the class **P** is closed under union, concatenation, and complement.

Notice that here the union, concatenation, and complement is for any possible language(s) in **P**. In other words, given $L_1$ and $L_2$ from **P**, you have to show that $L_1 \cup L_2$, $L_1 L_2$ and $\overline{L_1}$ is in **P**.

Recall that a language $L$ is in **P** if and only if there is a polynomial $f(n)$ and a Turing Machine $M$ such that, for any string $w$ of length $n$, $M$ will decide if $w \in L$ in time at most $f(n)$.

**Problem 2**: Show that the class **P** is closed under star operation. Hint: use dynamic programming. On input $y = y_1 y_2 \cdots y_n$ for $y_i \in \Sigma$, build a table indicating for each $i \leq j$, whether the substring $y_i y_{i+1} \cdots y_j \in A^*$ for some language $A \in$ **P**.

**Problem 3**: Let MODEXP= $\{\langle a, b, c, p\rangle \mid$ $a$, $b$, $c$, and $p$ are binary integers such that $a^b \equiv c \pmod p$). Show that MODEXP$\in$**P**.

Note that the most obivious algorithm does not run in polynomial time. Notice that the algorithm should run in polynomial of the input size, not the value of the input. For example, when given integer $a$, the value is $a$, but its input size is $\lceil \log a \rceil$.

Hint: Try it first when $b$ is a power of 2.

**Problem 4**: Let $D\text{-}SAT = \{\langle \phi \rangle \mid \phi$ has at least two satisfying assignments$\}$. Show that $D\text{-}SAT$ is NP-complete.

Hint: Prove by showing that $SAT$ can be reduced to this problem.

**Problem 5**: A subset $D$ of nodes of a undirected graph $G = (V, E)$ is a *dominating set* if every other node of $G$ is adjacent to some node in the subset $D$. Let problem

$$DOMINATING\_SET = \{\langle G, k\rangle \mid G \text{ has a dominating set with } k \text{ nodes}\}$$

A subset $C$ of nodes of an undirected graph $G$ is a *vertext cover* if every edge of $G$ is incident to one of the nodes in $C$.

$$VERTEX\_COVER = \{\langle G, k\rangle \mid G \text{ has a vertex cover with } k \text{ nodes}\}$$

Show that DOMINATING_SET is NP-complete by showing that VERTEX_COVER can be reduced to DOMINATING_SET.