CS 535 Design and Analysis of Algorithms **Fall Semester, 2021**

Extra set of problems

Assigned: Dec. 1 *Will not be collected/graded*

Variations of the following problems might appear on the final exam. No solutions for this set of problems will be posted.

Problem 1 Problem 24.1-3 from the textbook.

Problem 2 Assume that you *only* know that the problems from the handout are NP-complete. Consider the following decision problem, called BIN-PACKING: Given a list of positive integers (all numbers here written in binary) $x_1, ..., x_k$, an integer m, and another integer B, called *bin capacity*, is there a partition of $\{1, 2, \ldots, k\}$ into sets U_i, for $i = 1, 2, \ldots, m$, (and therefore $\bigcup_{i=1}^{m} U_i = \{1, 2, \ldots, k\}$) such that, for all $i = 1, 2, \ldots, m$, we have that $\sum_{j \in U_i} x_j \leq B$ (items packed in a bin do not exceed capacity). Prove that BIN-PACKING is NP-Complete.

Recommended but not required: Use the hardness of $SUBSET - SUM$.

Problem 3 A **multiple source-sink network** is a tuple $G = (V, E, c, S, T)$, where V is a set of vertices, E is a set of directed edges (parallel edges are allowed), $S \subset V$ is the set of **sources**, and $T \subset V$ is the set of **sinks**, c is a **capacity** function: $c : E \to Z_+$. Also, $S \cap T = \emptyset$. That is, sources are distinct from sinks.

A function $f : E \to R_+$ is called a *flow* if the following three conditions are satisfied:

1. *conservation of flow at interior vertices:* for all vertices u not in $S \cup T$,

$$\sum_{e \in \delta^-(u)} f(e) = \sum_{e \in \delta^+(u)} f(e) \ ;$$

2. *capacity constraints:* $f \leq c$ pointwise: i.e. for all $e \in E$,

$$f(e) \leq c(e) \ .$$

Assume that non-negative quantities p_s, for $s \in S$, and q_t, for $t \in T$, are given. The goal of this problem is to determine if a valid flow exists such that for all $s \in S$:

$$\sum_{e \in \delta^+(s)} f(e) - \sum_{e \in \delta^-(s)} f(e) = p_s$$

and such that for all $t \in T$:

$$\sum_{e \in \delta^-(t)} f(e) - \sum_{e \in \delta^+(t)} f(e) = q_t.$$

Use Network Flows to give a polynomial-time algorithm for this **decision** problem (the answer is YES or NO). Hint: read chapter 26.1 of the textbook.

Problem 4 Problem 22-2 from the textbook.