Extra set of problems

Assigned: Dec. 1 Will not be collected/graded

Variations of the following problems might appear on the final exam. No solutions for this set of problems will be posted.

Problem 1 Problem 24.1-3 from the textbook.

Problem 2 Assume that you only know that the problems from the handout are NP-complete. Consider the following decision problem, called BIN-PACKING: Given a list of positive integers (all numbers here written in binary) $x_1, \ldots, x_k$, an integer $m$, and another integer $B$, called bin capacity, is there a partition of $\{1, 2, \ldots, k\}$ into sets $U_i$, for $i = 1, 2, \ldots, m$, (and therefore $\bigcup_{i=1}^m U_i = \{1, 2, \ldots, k\}$) such that, for all $i = 1, 2, \ldots, m$, we have that $\sum_{j \in U_i} x_j \leq B$ (items packed in a bin do not exceed capacity). Prove that BIN-PACKING is NP-Complete.

Recommended but not required: Use the hardness of $\text{SUBSET} - \text{SUM}$.

Problem 3 A multiple source-sink network is a tuple $G = (V, E, c, S, T)$, where $V$ is a set of vertices, $E$ is a set of directed edges (parallel edges are allowed), $S \subset V$ is the set of sources, and $T \subset V$ is the set of sinks, $c$ is a capacity function: $c : E \rightarrow \mathbb{Z}_+$. Also, $S \cap T = \emptyset$. That is, sources are distinct from sinks.

A function $f : E \rightarrow \mathbb{R}_+$ is called a flow if the following three conditions are satisfied:

1. conservation of flow at interior vertices: for all vertices $u$ not in $S \cup T$,
   \[ \sum_{e \in \delta^-(u)} f(e) = \sum_{e \in \delta^+(u)} f(e) ; \]

2. capacity constraints: $f \leq c$ pointwise: i.e. for all $e \in E$,
   \[ f(e) \leq c(e) . \]

Assume that non-negative quantities $p_s$, for $s \in S$, and $q_t$, for $t \in T$, are given. The goal of this problem is to determine if a valid flow exists such that for all $s \in S$:

\[ \sum_{e \in \delta^+(s)} f(e) - \sum_{e \in \delta^-(s)} f(e) = p_s \]

and such that for all $t \in T$:

\[ \sum_{e \in \delta^-(t)} f(e) - \sum_{e \in \delta^+(t)} f(e) = q_t . \]

Use Network Flows to give a polynomial-time algorithm for this decision problem (the answer is YES or NO). Hint: read chapter 26.1 of the textbook.

Problem 4 Problem 22-2 from the textbook.