1. If the algorithm ever stops, it returns 1, but it may never stop!

Proof. By induction on $m$. For the base case, $m = 1$, clearly line 3 returns 1. For the inductive step, assume that $useless(1), \ldots, useless(m - 1)$ all return 1. If we ever have $random(1, m) = 1$ in line 5, $useless$ returns 1. If we never have $random(1, m) = 1$ in line 5, the algorithm never stops. \qed

2. The expected number of calls to $random$ in line 5 is $H_{m-1} + 1$, where $H_{m-1}$ is the $(m - 1)$st harmonic number. The analysis is very close to that of the quicksort.

Let the expected number be $f(m)$. Then because the random numbers are uniformly distributed, for $m \geq 2$ we have

$$f(m) = \frac{1 + f(m)}{m} + \frac{1 + f(m - 1)}{m} + \cdots + \frac{1 + f(2)}{m} + \frac{1}{m}$$

$$(m - 1)f(m) = m + \sum_{i=2}^{m-1} f(i)$$

(1)

Similarly, for $m \geq 3$ we have

$$f(m - 1) = \frac{1 + f(m - 1)}{m - 1} + \cdots + \frac{1 + f(2)}{m - 1} + \frac{1}{m - 1}$$

$$(m - 2)f(m - 1) = m - 1 + \sum_{i=2}^{m-2} f(i)$$

(2)

Subtracting equation (2) from equation (1), we obtain, for $m \geq 3$,

$$f(m) - f(m - 1) = \frac{1}{m - 1}$$

(3)

Summing (3), that is adding

$$f(m) - f(m - 1) = \frac{1}{m - 1}$$

$$f(m - 1) - f(m - 2) = \frac{1}{m - 2}$$

$$\cdots$$

$$f(3) - f(2) = 1,$$

gives

$$f(m) = \frac{1}{2} + \frac{1}{3} + \cdots + \frac{1}{m - 1} + f(2).$$

(4)
But $f(1) = 0$ (why?) and $1$ gives $f(2) = 2$, so for $m > 1$ we have

$$f(m) = H_{m-1} + 1.$$ 

3. The worst-case is when $\text{random}$ never returns 1, and $\text{useless}$ never stops; in that case, the number of calls to $\text{random}$ is infinite.